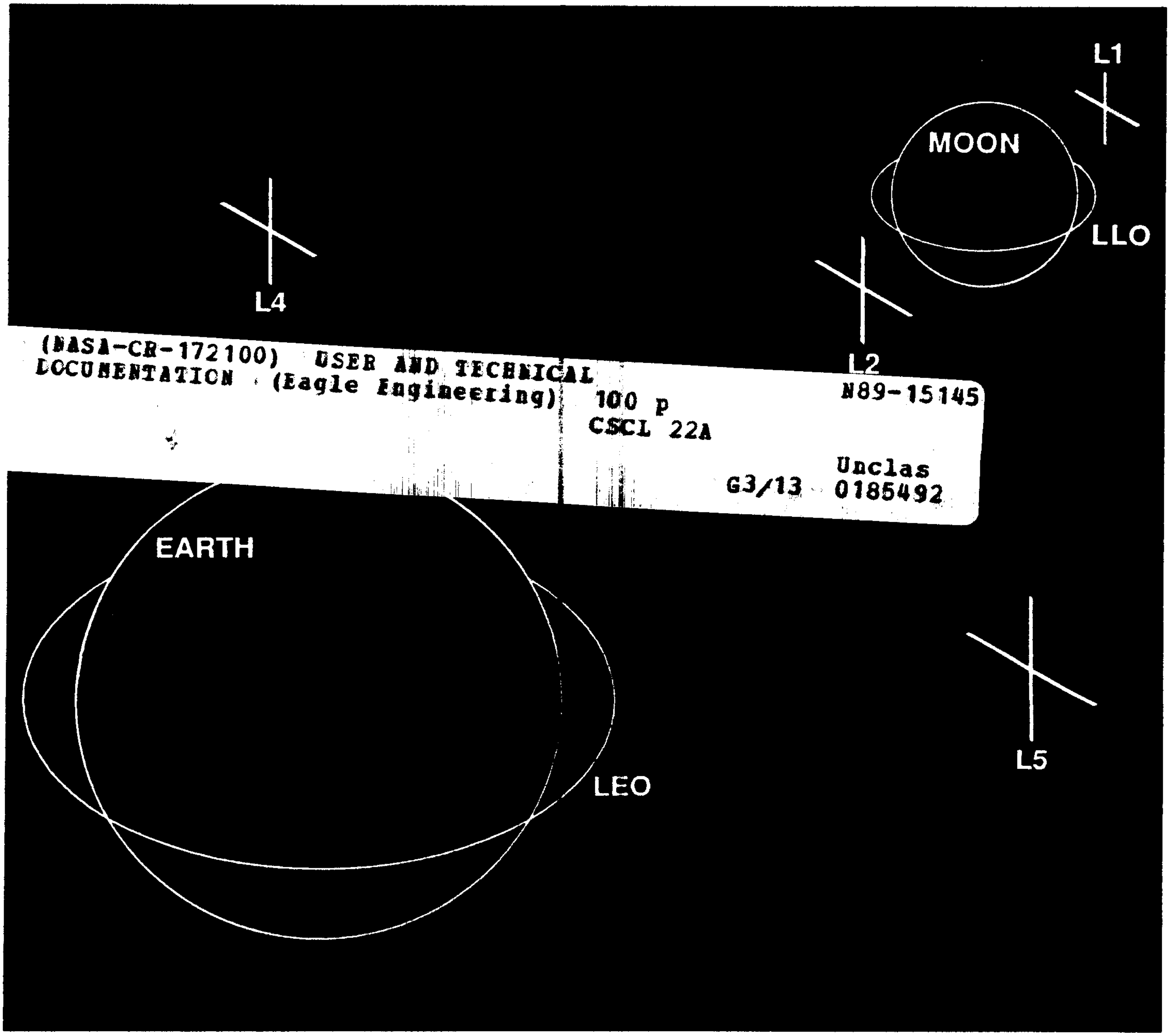
CR 172100

LBSS

LEO to L1 Trajectory Program



NASA Contract No. NAS9-17878
Eagle Eng. Report No. 88-219
October 30, 1988

EAGLE

LP1

User and Technical Documentation

National Aeronautics and Space Administration
Lyndon B. Johnson Space Center

Advanced Projects Office

Eagle Engineering, Inc.
Houston, Texas
October, 1988

NASA Contract NAS9-17878
Eagle Engineering Report No. 88-219

Foreword

This program is an important tool in the study of alternative routes between the Earth and the Moon. Dr. John Alred was the NASA Technical Monitor for the contract under which this program was produced. Mr. Andy Petro was the NASA Task Manager for this particular task. Mr. Bill Stump was the Eagle Project Manager for the contract under which this program was produced. The program was written by Jack Funk, originally in Quick Basic, and translated into Fortran by Mr. Bill Englom. Mr. Englom also prepared the documentation.

Table of Contents

1.0 Introduction

The program LP1 calculates outbound and return trajectories between low earth orbit (LEO) and libration point #1 (L1). Libration points (LP) are defined as locations in space that orbit the Earth such that they are always stationary with respect to the Earth-Moon line. L1 is located behind the Moon such that the pull of the Earth and Moon together just cancel the centrifugal acceleration associated with the libration point's orbit.

The outbound flights depart from a circular orbit of any altitude and inclination about the Earth and culminate in a circular orbit about the Earth at libration point #1 within a specified flight time. The flight involves three burns.

First, the departure orbit is made into a more eccentric orbit (ellipse or hyperbola) with an initial ΔV in order to reach the lunar sphere of influence (SOI), a region where the vehicle is near its lowest velocity in the trajectory. The SOI is a spherical region whose surface normally includes all the points at a distance of 11% of the Earth-Moon distance from the Moon's center. However, in order to simplify the calculations this radius was increased to include L1, enlarging the sphere radius to 15% of the Earth-Moon distance. Note, this change in SOI radius should not change the results significantly. A given flight may penetrate the SOI at a number of points identified by projecting lunar latitude and longitude onto the SOI. For each flight the program will calculate a set of possible trajectories associated with a set of SOI penetration points -- a matrix of longitudes and latitudes.

Next, a second burn (at the SOI) is performed involving a "flyby" of the Moon from the SOI point above the front side of the Moon to L1 behind the back side. Note, the SOI penetration point and L1 will always be the same distance from the center of the Moon. From the geometry of the trajectory it is apparent that the Lunar "flyby" perigee altitude, supplied by the user, will occur midway between the SOI point and L1. Consequently, the geometry of the orbit will force true anomaly, flight path angle, and absolute time to or from perigee passage to be the same for both the SOI and L1 points. There are two paths between these points, posigrade and retrograde. LP1 calculates only posigrade "flyby" trajectories since retrograde orbits that pass through the perigee altitude are not always possible while there is always a posigrade solution. The retrograde trajectories warrant more study in future work. Since L1 is constantly rotating with the Moon, this trajectory is iterated until L1 is reached.

The third burn is simply a circularization of the trajectory at L1 about the Earth. The velocity vector is corrected to that of L1. Note, once the SOI to L1 trajectory has been established, the Earth-SOI flight is iterated until the total transfer time, including the transfers from LEO to the SOI, and SOI to L1, match the user's flight time constraint (an input value). This is done for a matrix of SOI penetration points, as mentioned earlier.

The return trajectories, which start at L1 and finish in the specified LEO orbit within the specified flight time, are calculated similarly. For instance, the "flyby" trajectory is calculated first, starting at L1 and finishing at the SOI via a posigrade orbit calculated using the same geometric simplifications described above. Note, the flight profile used in this program to calculate flights between Earth and L1 may not be the optimum with respect to a minimum ΔV.

After the user has defined the trajectory as outbound or return, the Earth orbit altitude and inclination, and the total flight time, LP1 produces matrices which display the total ΔVs, the three component ΔVs (described above), the "fly-by" trajectory inclinations, and the "fly-by" azimuth angle at the SOI for the resulting flight from Earth to L1 for a representative set of SOI points. These points are defined by the user, who provides a starting longitude and latitude, and an increment for each. The matrix is built with 10 longitudes forming the columns and 19 latitudes forming the rows.

Section 2.0 of this document describes the input required from the user to define the flight. Section 3.0 describes the contents of the six reports that are produced as outputs. Section 4.0 includes the instructions needed to execute the program.

A more detailed description of the process used in LP1 has been included as Appendix D (main program), Appendix E (in-program subroutine), and Appendices F, G, H, I, and J (external subroutines). LP1 was derived from the PLANECHG program (also produced under this contract) with the major addition of the FLYBY subroutine. Therefore, the documentation for PLANECHG may be used as a reference for many of the equations, variables, and conventions used in LP1 (except in the FLYBY routine).

2.0 Program Inputs

The following paragraphs discuss the inputs provided by the user. The prompt is the message displayed by the program onto the screen. The input variable is the program variable assigned to the user's response. The description provides information about how to respond to the prompt.

1. Prompt: INPUT OUTBOUND OR RETURN

   Input variable: MD

   Description: Enter OUTBOUND for Earth-to-L1 trajectory calculations. Enter RETURN for L1-to-Earth trajectory calculations.

2. Prompt: INPUT PERIGEE ALTITUDE OF EARTH ORBIT (NMI)

   Input variable: HPE

   Description: Enter the height above the Earth's surface of the Earth circular orbit, in nautical miles.

3. Prompt: INPUT PERIGEE ALTITUDE OF LUNAR ORBIT (NMI)

   Input variable: HPM

   Description: Enter the height above the Lunar surface of the Lunar circular orbit, in nautical miles.

4. Prompt: INPUT EARTH DEPARTURE JULIAN DATE

Input variable: TIMJ

Description: Enter the origin trans-SOI injection date (Earth departure date for outbound trajectories) in Julian day format, where January 1, 2000 is day 2,451,545. Refer to Section C of "The Astronomical Almanac of the Year 1988". Day 2451545 is the default value if zero is entered in this field.

5. Prompt: INITIAL LONGITUDE

Input variable: ALONI

Description: Enter initial sphere of influence longitude for the output matrices. This value will become the heading for column 1 of the matrices.

6. Prompt: INPUT INCREMENT FOR THE MAP

Input variable: DELLON

Description: Enter longitude increments for the output matrices. Applied to the initial longitude, this value defines the subsequent column headings of the matrices. Longitudes for outbound trajectories should be between zero and -90 degrees.

7. Prompt: INPUT INITIAL LATITUDE

Input variable: ALATI

Description: Enter initial sphere of influence latitude for the output matrices. This value will become the heading for row 1 of the matrices.

8. Prompt: INPUT INCREMENT FOR THE MAP

Input variable: DELLAT

Description: Enter latitude increments for the output matrices. Applied to the initial latitude, this value defines the subsequent row headings of the matrices.

9. Prompt: INPUT EARTH ORBIT TO LUNAR ORBIT INCLINATION

Input variable: AINCEO

Description: Enter Earth circular orbit inclination, in degrees. This is not what is typically considered inclination (i.e., a measurement taken from the Earth's equatorial plane), but rather the angle between the plane of the low Earth orbit and the plane of the Moon's orbit about the Earth.

10. Prompt: INPUT FLIGHT TIME

Input variable: FTIM

Description: Enter the desired total flight time from LEO to L1, in hours.

3.0 Program Outputs

This section describes the contents of each of the six reports generated by the program. These reports may be found in the output file, LP1.OUT. Samples of all six reports have also been included.

Report #1: Total Delta Velocity Map For Outbound/Return Trajectories

The top section of this report repeats the input values entered by the user. The second section is a 10 X 19 matrix of total velocities (in ft/sec) required to fly the profile described by the inputs. Each cell corresponds to a particular latitude and longitude on the Lunar sphere of influence. A "flyby" burn (transfer from SOI to L1) may occur at any one of these coordinates, and the value of the corresponding cell is the total velocity required for the flight if the "flyby" burn occurs at that location. The total is the sum of the Earth-SOI transfer orbit injection ΔV (from LEO to SOI), the sphere of influence to L1, Lunar "flyby" ΔV (from SOI to L1), and the destination circular orbit injection ΔV (circularization at L1). Note, the longitudes must always be between 0° and -90° for outbound trajectories, and between 0° and +90° for return trajectories. The third section of the report is a summary of key data corresponding to the matrix cell containing the lowest total velocity. This data includes:

- X-, Y-, and Z-components of velocity at the sphere of influence just before and just after the "flyby" burn (VX, VY, VZ).
- Total magnitude of the velocity at the sphere of influence just before and just after the "flyby" burn (VEL).
- Flight path angle at the sphere of influence just before and just after the "flyby" burn (GAMA).
- Azimuth of the sphere of influence point from the Earth and from the Moon (AZM).

- Earth and "flyby" orbit inclinations measured from the Earth-Moon plane (AINC).

- Earth-SOI transfer trajectory and "flyby" orbit insertion ascending/descending node positions measured from Earth-Moon line at 0° longitude (ANODE).

- Earth-to-SOI and SOI-to-L1 times of flight (TIME).

- Earth-SOI transfer trajectory and final L1 orbit insertion ΔV's (DVCIR).

- Sphere of influence, Lunar "flyby" ΔV (DVPHER).

- Total ΔV (DVTOTAL).

VELOCITY MAP FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE OPTION 0
DATE 7-NOV-88 TIME 14:46:12

JUILIAN DAY 2451545. PERIGEE ALT (NMI) EARTH = 250. MOON = 10000.
TRANSLUNAR FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON PAGE3

ALON> / ALAT	0.0	-5.0	-10.0	-15.0	-20.0	-25.0	-30.0	-35.0	-40.0	-45.0
90.0	14701.	14819.	14862.	14902.	14938.	14970.	14998.	15023.	15043.	15059.
80.0	13866.	13863.	13863.	13866.	13872.	13881.	13893.	13907.	13924.	13943.
70.0	13690.	13672.	13663.	13661.	13667.	13682.	13705.	13737.	13778.	13826.
60.0	0.	13542.	13508.	13488.	13485.	13499.	13532.	13585.	13657.	13746.
50.0	0.	13493.	13420.	13367.	13341.	13345.	13383.	13457.	13568.	13713.
40.0	0.	0.	0.	13314.	13248.	13229.	13264.	13358.	13513.	13729.
30.0	0.	0.	0.	0.	13211.	13157.	13180.	13290.	13492.	13790.
20.0	0.	0.	0.	0.	0.	13126.	13130.	13248.	13492.	13876.
10.0	0.	0.	0.	0.	0.	13125.	13111.	13231.	13505.	13953.
0.0	0.	0.	0.	0.	0.	13143.	13118.	13234.	13514.	13985.
-10.0	0.	0.	0.	0.	0.	13173.	13147.	13255.	13516.	13955.
-20.0	0.	0.	0.	0.	0.	13216.	13198.	13294.	13517.	13884.
-30.0	0.	0.	0.	0.	13352.	13276.	13273.	13355.	13532.	13809.
-40.0	0.	0.	0.	13491.	13406.	13363.	13372.	13439.	13570.	13763.
-50.0	0.	13696.	13610.	13542.	13497.	13480.	13497.	13549.	13639.	13765.
-60.0	0.	13723.	13677.	13645.	13627.	13627.	13644.	13681.	13738.	13813.
-70.0	13844.	13820.	13804.	13794.	13791.	13797.	13811.	13834.	13865.	13904.
-80.0	13984.	13978.	13975.	13975.	13977.	13982.	13989.	14000.	14012.	14028.
-90.0	14836.	14881.	14923.	14962.	14998.	15030.	15059.	15083.	15104.	15121.

MINIMUM VELOCITY PRINT NODE OPTION 0
DATE 7-NOV-88 TIME 14:46:12

FLIGHT TIME = 200. LAT = 10.0 LON = -30.0 LUNAR AINC = 10.0 EARTH AINC = 25.0

BODY	VX	VY	VZ	VEL	GAMA	AZM	AINC	ANODE	TIME	DVCIR		
MOON	100.	2304.	-96.	900.	-38.3	90.0	10.0	240.3	109.8	1402.	DVPHER =	1679.
EARTH	447.	672.	-279.	853.	35.3	114.9	25.0	8.5	90.2	10030.	DVTOTAL =	13111.

Report #2

Map of Delta Velocity at Sphere of Influence for Outbound/Return Trajectories

This report is a matrix of the delta velocities that occur at the sphere of influence to match the velocity vectors of the Earth-to SOI trajectory and the SOI-to-L1 "flyby" trajectory for each SOI point. Each cell corresponds to a particular latitude and longitude on the sphere of influence at which a burn may occur.

MAP OF DELTA VELOCITY AT SPHERE OF INFLUENCE FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE OPTION 0

DATE 7-NOV-88 TIME 14:46:12

EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON = 90.0

ALON> ALAT	0.0	-5.0	-10.0	-15.0	-20.0	-25.0	-30.0	-35.0	-40.0	-45.0
90.0	3372.1	3478.4	3521.5	3560.9	3596.8	3629.0	3657.4	3681.9	3702.3	3718.6
80.0	2485.8	2486.1	2489.7	2496.4	2506.0	2518.5	2533.9	2552.1	2572.8	2595.9
70.0	2264.0	2254.5	2253.2	2259.7	2274.5	2297.8	2329.9	2370.5	2419.1	2475.0
60.0	0.0	2077.7	2058.3	2053.8	2065.4	2094.5	2142.1	2208.2	2292.3	2393.0
50.0	0.0	1977.6	1928.1	1899.1	1895.5	1921.4	1980.2	2073.2	2200.1	2359.0
40.0	0.0	0.0	0.0	1810.6	1777.6	1788.4	1850.4	1968.2	2143.2	2373.7
30.0	0.0	0.0	0.0	0.0	1716.9	1701.7	1758.0	1896.1	2120.8	2433.7
20.0	0.0	0.0	0.0	0.0	0.0	1658.8	1701.8	1852.3	2120.8	2518.1
10.0	0.0	0.0	0.0	0.0	0.0	1649.9	1678.9	1834.6	2133.3	2592.6
0.0	0.0	0.0	0.0	0.0	0.0	1665.3	1684.4	1836.8	2142.4	2623.1
-10.0	0.0	0.0	0.0	0.0	0.0	1698.0	1715.2	1858.0	2145.0	2594.9
-20.0	0.0	0.0	0.0	0.0	0.0	1748.2	1770.2	1898.1	2145.9	2526.2
-30.0	0.0	0.0	0.0	0.0	1858.1	1820.5	1850.7	1961.6	2161.1	2452.8
-40.0	0.0	0.0	0.0	1987.8	1934.9	1922.1	1958.2	2049.5	2199.5	2408.2
-50.0	0.0	2180.8	2118.3	2073.5	2051.4	2056.9	2093.8	2164.8	2270.7	2410.4
-60.0	0.0	2258.0	2227.6	2210.7	2208.4	2222.5	2254.4	2304.8	2373.6	2459.8
-70.0	2418.2	2402.3	2393.9	2392.5	2398.8	2413.1	2435.9	2467.0	2506.3	2553.2
-80.0	2604.1	2601.0	2601.5	2604.7	2610.6	2619.3	2630.8	2644.9	2661.6	2680.6
-90.0	3495.4	3540.2	3582.3	3621.4	3657.0	3689.2	3717.7	3742.4	3763.2	3780.0

Report #3

Map of Inclinations of Flyby Orbits for Outbound/Return Trajectories

This report contains a matrix of the inclinations of the "flyby" trajectories with respect to the Earth-Moon plane, between each SOI point and L1. Each cell corresponds to a particular latitude and longitude on the sphere of influence at which the trajectory originates or culminates.

MAP OF INCLINATION OF FLY-BY ORBIT FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE OPTION

0

DATE 7-NOV-88 TIME 14:46:12

EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON = 90.0

ALON> ALAT	0.0	-5.0	-10.0	-15.0	-20.0	-25.0	-30.0	-35.0	-40.0	-45.0
90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0
80.0	81.5	81.0	80.7	80.4	80.2	80.0	80.0	80.0	80.2	80.4
70.0	73.2	72.3	71.5	70.8	70.4	70.1	70.0	70.1	70.4	70.9
60.0	0.0	63.6	62.3	61.3	60.6	60.1	60.0	60.2	60.6	61.4
50.0	0.0	54.8	53.1	51.7	50.7	50.2	50.0	50.2	50.8	51.8
40.0	0.0	0.0	0.0	42.0	40.9	40.2	40.0	40.2	41.0	42.2
30.0	0.0	0.0	0.0	0.0	30.8	30.2	30.0	30.2	31.0	32.2
20.0	0.0	0.0	0.0	0.0	0.0	20.2	20.0	20.2	20.8	21.9
10.0	0.0	0.0	0.0	0.0	0.0	10.1	10.0	10.1	10.5	11.1
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
-10.0	0.0	0.0	0.0	0.0	0.0	10.1	10.0	10.1	10.5	11.1
-20.0	0.0	0.0	0.0	0.0	0.0	20.2	20.0	20.2	20.8	21.9
-30.0	0.0	0.0	0.0	0.0	30.8	30.2	30.0	30.2	31.0	32.2
-40.0	0.0	0.0	0.0	42.0	40.9	40.2	40.0	40.2	41.0	42.2
-50.0	0.0	54.8	53.1	51.7	50.7	50.2	50.0	50.2	50.8	51.8
-60.0	0.0	63.6	62.3	61.3	60.6	60.1	60.0	60.2	60.6	61.4
-70.0	73.2	72.3	71.5	70.8	70.4	70.1	70.0	70.1	70.4	70.9
-80.0	81.5	81.0	80.7	80.4	80.2	80.0	80.0	80.0	80.2	80.4
-90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0	90.0

Report #4

Map of Delta Velocity at L1 for Outbound/Return Trajectories

This report is a matrix of the delta velocities that occur at L1 to circularize the trajectory at L1 (outbound flights), or to initiate a posigrade "flyby" trajectory from L1 to a particular SOI point (return flights). Each cell corresponds to a particular latitude and longitude on the sphere of influence through which the "flyby" trajectory originates or culminates.

MAP OF DELTA VELOCITY AT L1 FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORIES NODE OPTION 0

DATE 7-NOV-88 TIME 14:46:12

EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON = 90.0

ALON> ALAT	0.0	-5.0	-10.0	-15.0	-20.0	-25.0	-30.0	-35.0	-40.0	-45.0
90.0	1233.	1241.	1241.	1241.	1241.	1241.	1241.	1241.	1241.	1241.
80.0	1301.	1296.	1292.	1287.	1282.	1277.	1271.	1265.	1259.	1253.
70.0	1363.	1354.	1344.	1334.	1323.	1311.	1299.	1287.	1274.	1262.
60.0	0.	1414.	1398.	1380.	1362.	1344.	1325.	1306.	1287.	1268.
50.0	0.	1476.	1451.	1426.	1400.	1374.	1348.	1322.	1297.	1272.
40.0	0.	0.	0.	1469.	1434.	1401.	1368.	1336.	1304.	1274.
30.0	0.	0.	0.	0.	1464.	1423.	1383.	1345.	1309.	1274.
20.0	0.	0.	0.	0.	0.	1440.	1395.	1352.	1312.	1273.
10.0	0.	0.	0.	0.	0.	1450.	1402.	1356.	1313.	1273.
0.0	0.	0.	0.	0.	0.	1454.	1404.	1357.	1314.	1273.
-10.0	0.	0.	0.	0.	0.	1450.	1402.	1356.	1313.	1273.
-20.0	0.	0.	0.	0.	0.	1440.	1395.	1352.	1312.	1273.
-30.0	0.	0.	0.	0.	1464.	1423.	1383.	1345.	1309.	1274.
-40.0	0.	0.	0.	1469.	1434.	1401.	1368.	1336.	1304.	1274.
-50.0	0.	1476.	1451.	1426.	1400.	1374.	1348.	1322.	1297.	1272.
-60.0	0.	1414.	1398.	1380.	1362.	1344.	1325.	1306.	1287.	1268.
-70.0	1363.	1354.	1344.	1334.	1323.	1311.	1299.	1287.	1274.	1262.
-80.0	1301.	1296.	1292.	1287.	1282.	1277.	1271.	1265.	1259.	1253.
-90.0	1241.	1241.	1241.	1241.	1241.	1241.	1241.	1241.	1241.	1241.

Report #5

Map of Delta Velocity at Earth for Outbound/Return Trajectories

This report is a matrix of the delta velocities that occur to initiate a transfer orbit from LEO to the SOI. Each cell corresponds to a particular latitude and longitude on the sphere of influence at which a burn may occur.

DELTA VELOCITY AT EARTH FOR EARTH TO L1 FLYBY TRANSFER TRAJECTORY NODE OPTION 0
DATE 7-NOV-88 TIME 14:46:12
EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON = 90.0

ALON> ALAT	0.0	-5.0	-10.0	-15.0	-20.0	-25.0	-30.0	-35.0	-40.0	-45.0
90.0	10096.	10100.	10100.	10100.	10100.	10100.	10100.	10100.	10100.	10100.
80.0	10080.	10081.	10082.	10083.	10084.	10086.	10088.	10090.	10092.	10094.
70.0	10063.	10064.	10065.	10067.	10070.	10073.	10076.	10080.	10084.	10089.
60.0	0.	10050.	10052.	10054.	10057.	10060.	10065.	10070.	10077.	10085.
50.0	0.	10040.	10041.	10043.	10045.	10049.	10055.	10062.	10071.	10082.
40.0	0.	0.	0.	10034.	10036.	10040.	10046.	10054.	10066.	10081.
30.0	0.	0.	0.	0.	10029.	10033.	10039.	10048.	10062.	10082.
20.0	0.	0.	0.	0.	0.	10028.	10033.	10043.	10059.	10085.
10.0	0.	0.	0.	0.	0.	10024.	10030.	10041.	10058.	10088.
0.0	0.	0.	0.	0.	0.	10023.	10029.	10040.	10058.	10089.
-10.0	0.	0.	0.	0.	0.	10024.	10030.	10041.	10058.	10088.
-20.0	0.	0.	0.	0.	0.	10028.	10033.	10043.	10059.	10085.
-30.0	0.	0.	0.	0.	10029.	10033.	10039.	10048.	10062.	10082.
-40.0	0.	0.	0.	10034.	10036.	10040.	10046.	10054.	10066.	10081.
-50.0	0.	10040.	10041.	10043.	10045.	10049.	10055.	10062.	10071.	10082.
-60.0	0.	10050.	10052.	10054.	10057.	10060.	10065.	10070.	10077.	10085.
-70.0	10063.	10064.	10065.	10067.	10070.	10073.	10076.	10080.	10084.	10089.
-80.0	10080.	10081.	10082.	10083.	10084.	10086.	10088.	10090.	10092.	10094.
-90.0	10100.	10100.	10100.	10100.	10100.	10100.	10100.	10100.	10100.	10100.

Report #6

Map of Azimuths of Flyby Orbits at SOI for Outbound/Return Trajectories

This report contains a matrix of the azimuth angles of the "flyby" trajectories at the SOI for each particular SOI point. Each cell corresponds to a particular latitude and longitude on the sphere of influence through which the "flyby" trajectory originates or culminates.

MAP OF AZMM FOR EARTH TO L1 FLYBY TRANSFERTRAJECTORY NODE OPTION 0
DATE 7-NOV-88 TIME 14:46:12
EARTH TO L1 FLYBY TRANSFER FLIGHT TIME (HR) = 200.0 INCL EARTH 25.0 INCL MOON = 90.0

ALON> ALAT	0.0	-5.0	-10.0	-15.0	-20.0	-25.0	-30.0	-35.0	-40.0	-45.0
90.0	180.	180.	180.	180.	180.	180.	180.	180.	180.	180.
80.0	121.	116.	111.	106.	100.	95.	90.	84.	79.	74.
70.0	122.	117.	112.	106.	101.	95.	90.	84.	79.	73.
60.0	0.	117.	112.	106.	101.	95.	90.	84.	79.	73.
50.0	0.	116.	111.	105.	100.	95.	90.	85.	79.	74.
40.0	0.	0.	0.	104.	99.	94.	90.	85.	80.	75.
30.0	0.	0.	0.	0.	98.	94.	90.	86.	82.	78.
20.0	0.	0.	0.	0.	0.	93.	90.	87.	84.	81.
10.0	0.	0.	0.	0.	0.	91.	90.	88.	87.	85.
0.0	0.	0.	0.	0.	0.	90.	90.	90.	90.	90.
-10.0	0.	0.	0.	0.	0.	89.	90.	92.	93.	95.
-20.0	0.	0.	0.	0.	0.	87.	90.	93.	96.	99.
-30.0	0.	0.	0.	0.	82.	86.	90.	94.	98.	102.
-40.0	0.	0.	0.	76.	81.	86.	90.	95.	100.	105.
-50.0	0.	64.	69.	75.	80.	85.	90.	95.	101.	106.
-60.0	0.	63.	68.	74.	79.	85.	90.	96.	101.	107.
-70.0	58.	63.	68.	74.	79.	85.	90.	96.	101.	107.
-80.0	59.	64.	69.	74.	80.	85.	90.	96.	101.	106.
-90.0	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.

4.0 Program Execution Instructions

The following instructions describe the steps to be taken by the user to execute this program:

A. Obtain access to the DEC VAX minicomputer and sign on with user identification.

B. At the $ prompt, type <u>RUN LP1</u>.

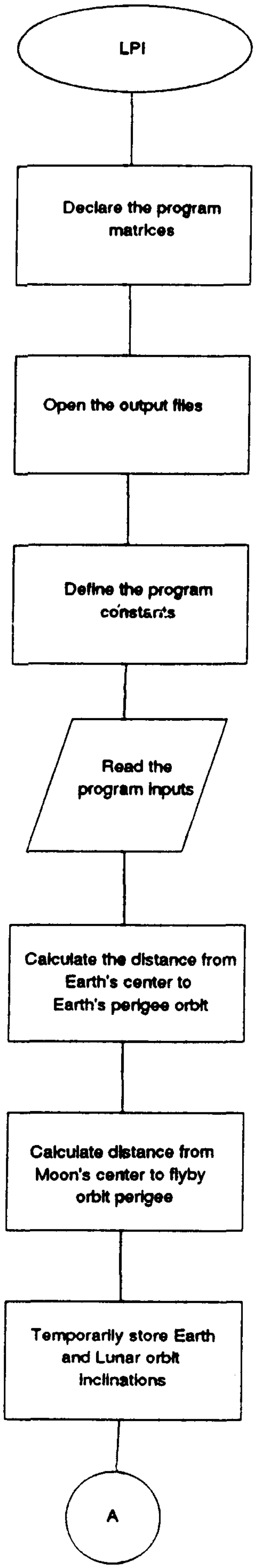
C. When prompted by the program, enter the program inputs. See section 2.0 for a discussion of the inputs.

D. After the last input has been entered, the program will execute for approximately 5 minutes, during which comments will appear notifying the user which trajectory, by SOI latitude and longitude, is currently being calculated. Upon completion, the message FORTRAN STOP will appear, followed by the $ prompt.

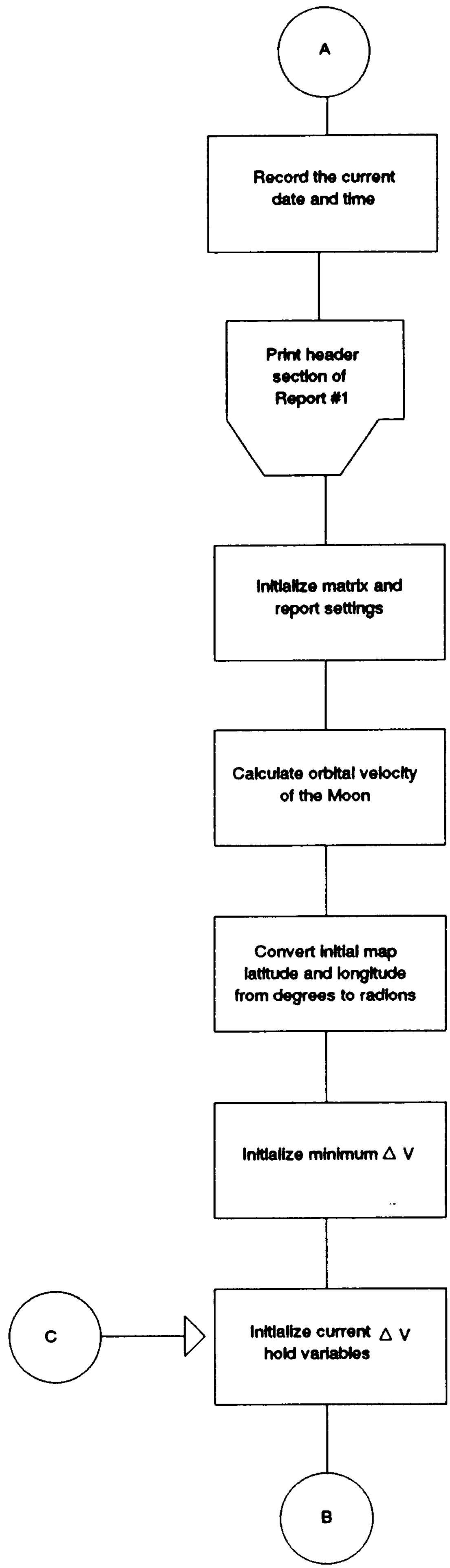
E. The program outputs will be placed in a file named LIBRATE.OUT;### where ### is a system generated number of the report. To print the most recently generated report, type the following at the $ prompt: <u>TYPE LP1.OUT</u>

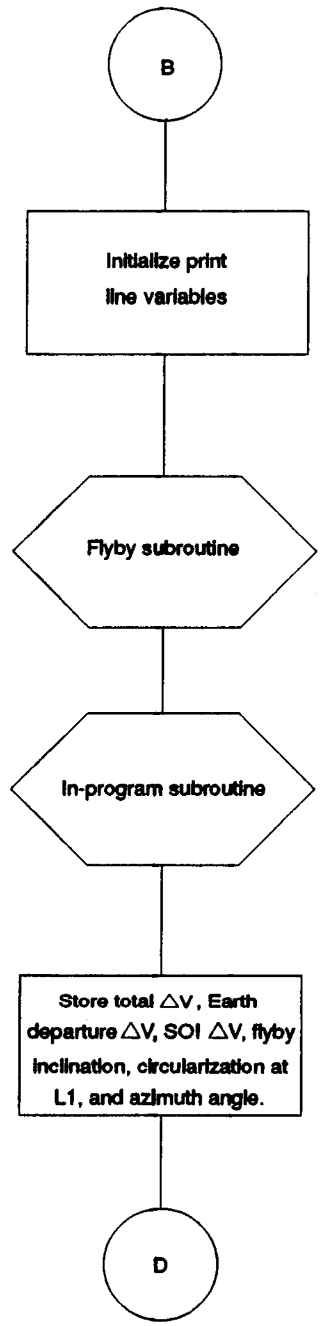
F. To re-execute the program with new parameters, begin again at step (B) above.

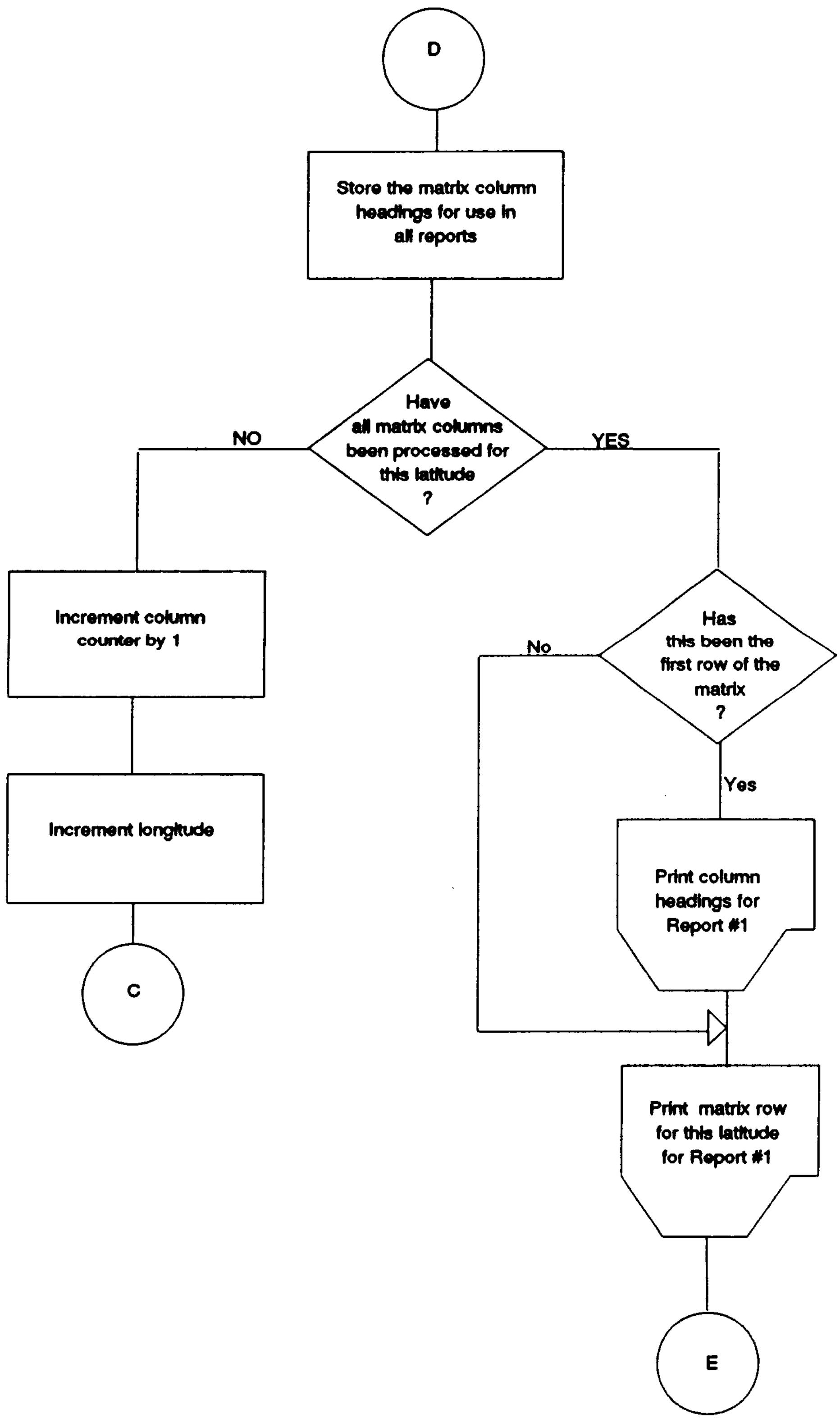
Appendix A. Program Flow Chart

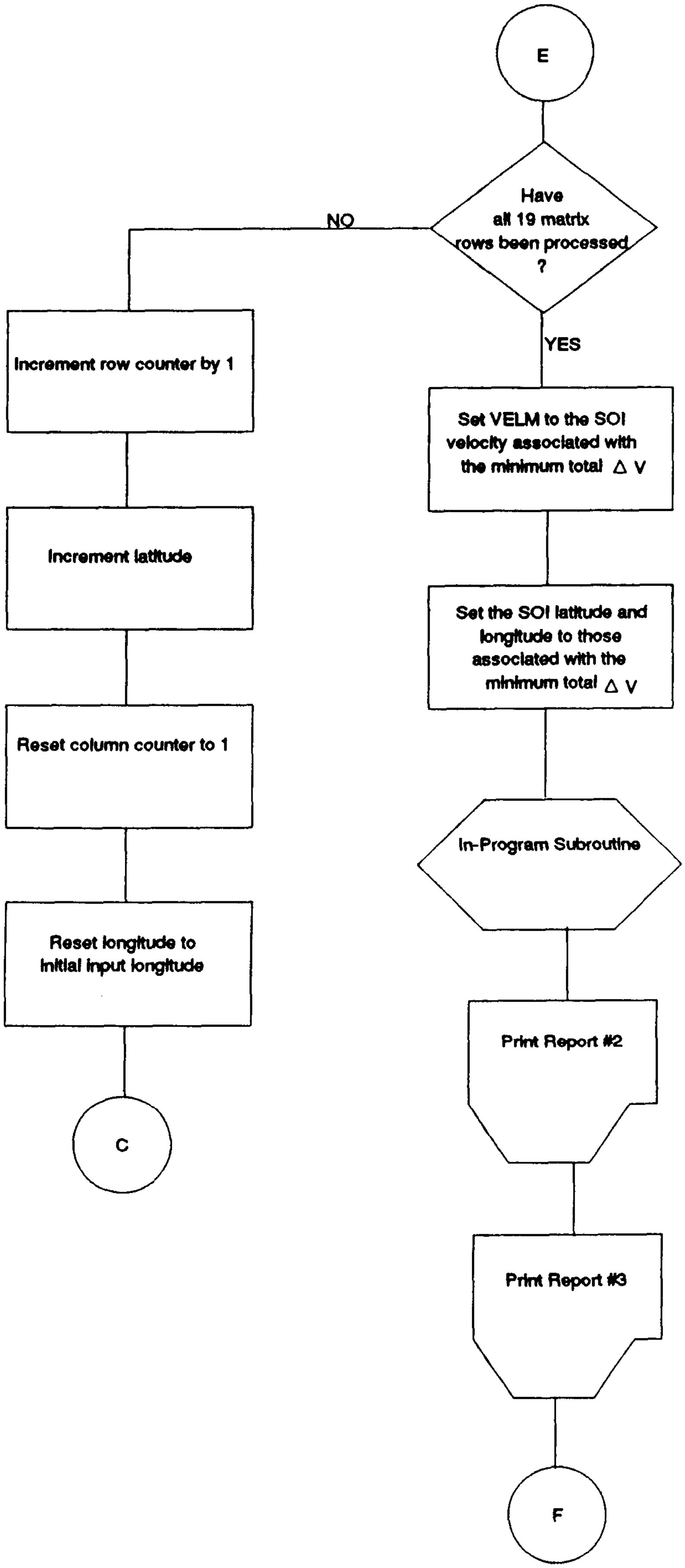


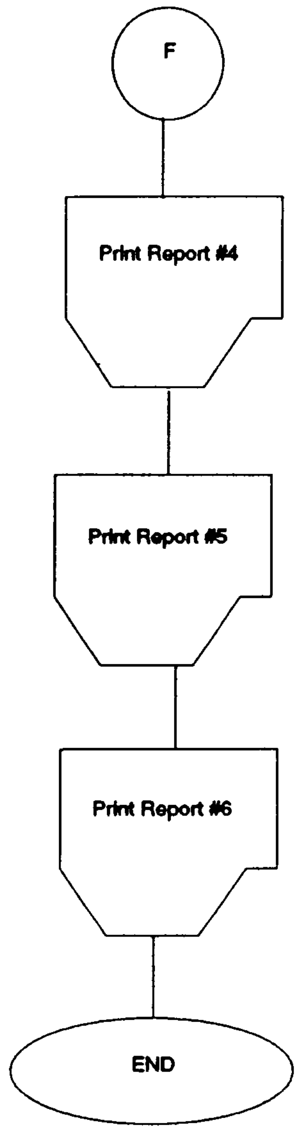
LPI
Declare the program matrices
Open the output files
Define the program constants
Read the program inputs
Calculate the distance from Earth's center to Earth's perigee orbit
Calculate distance from Moon's center to flyby orbit perigee
Temporarily store Earth and Lunar orbit inclinations
A

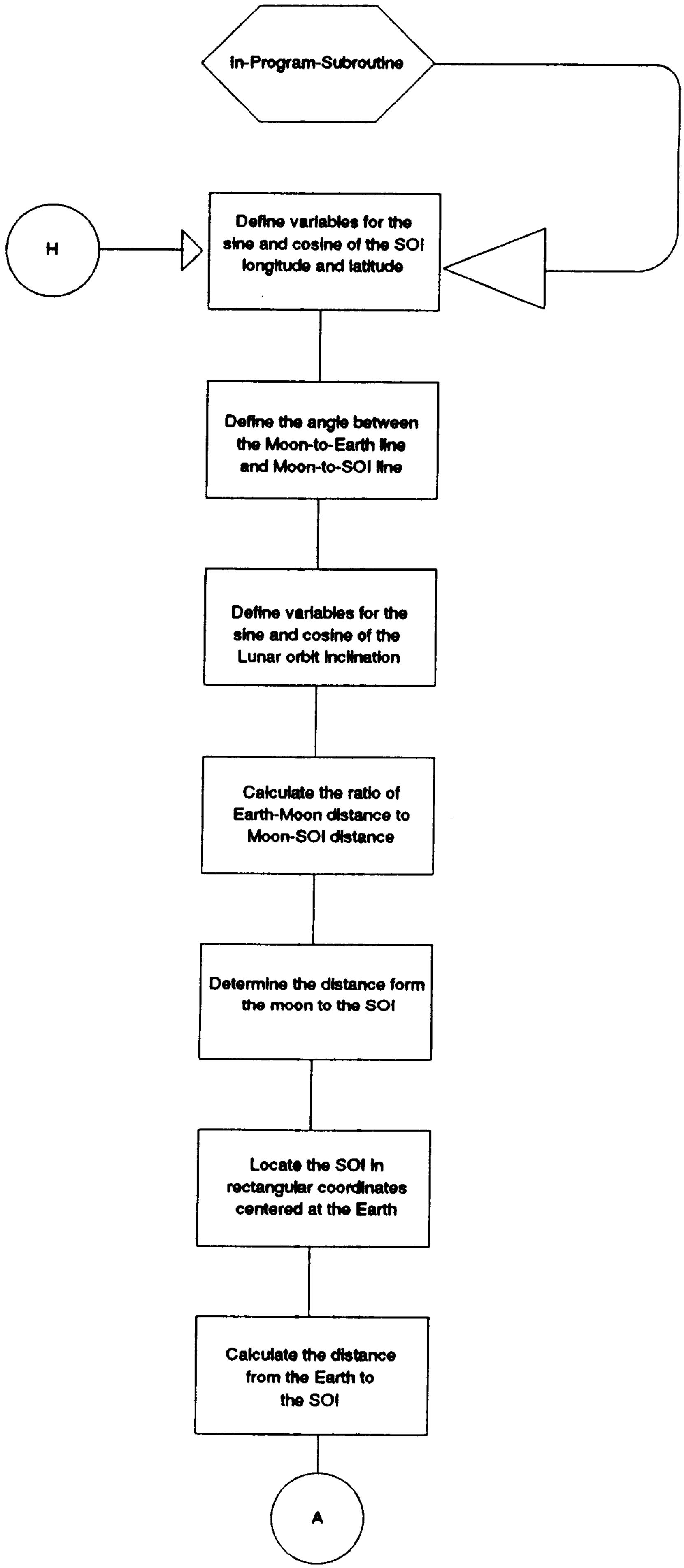

A
Record the current date and time
Print header section of Report #1
Initialize matrix and report settings
Calculate orbital velocity of the Moon
Convert initial map latitude and longitude from degrees to radions
Initialize minimum Δ V
C
Initialize current Δ V hold variables
B

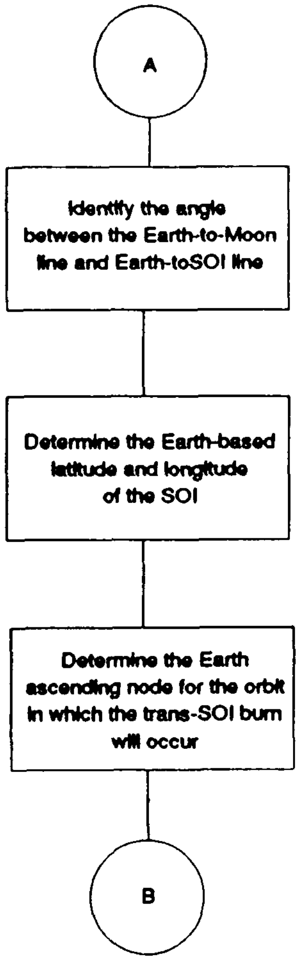

B
Initialize print line variables
Flyby subroutine
In-program subroutine
Store total △V, Earth departure △V, SOI △V, flyby inclination, circularization at L1, and azimuth angle.
D

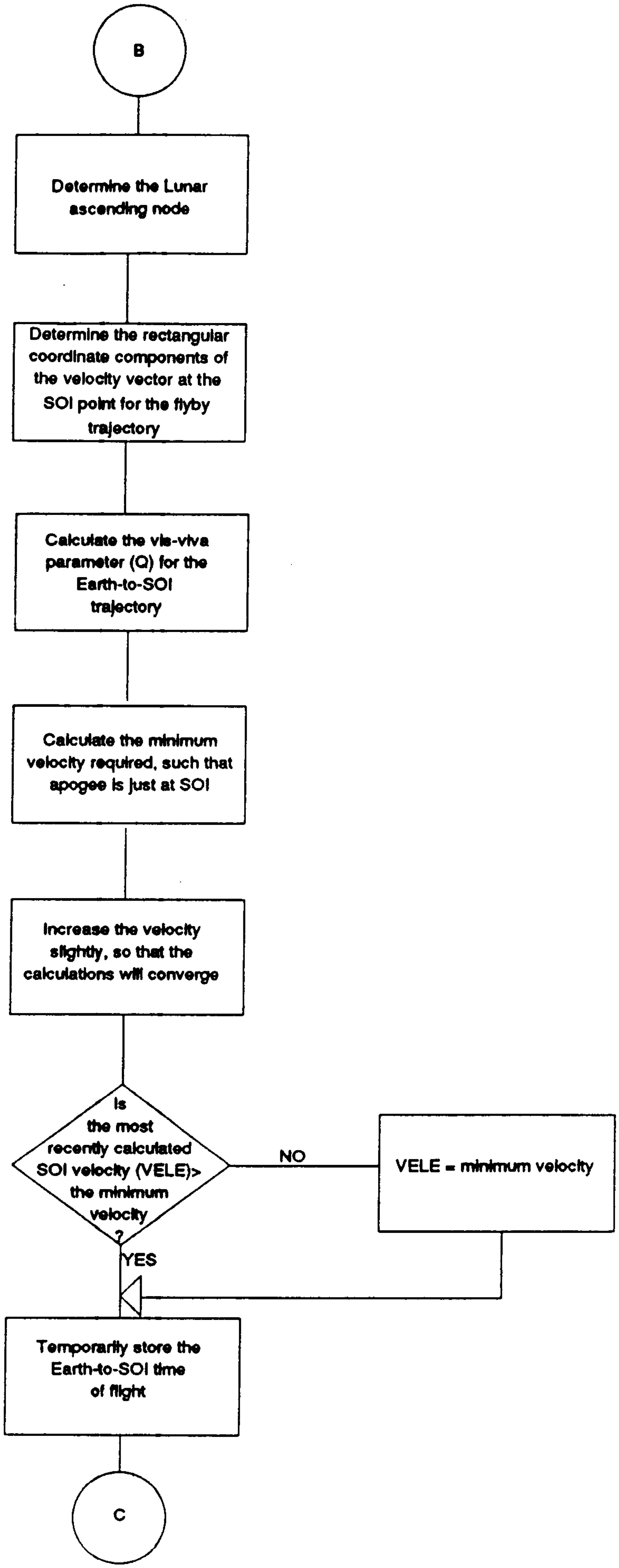

D
Store the matrix column headings for use in all reports
Have all matrix columns been processed for this latitude ?
NO
YES
Increment column counter by 1
Increment longitude
C
Has this been the first row of the matrix ?
No
Yes
Print column headings for Report #1
Print matrix row for this latitude for Report #1
E

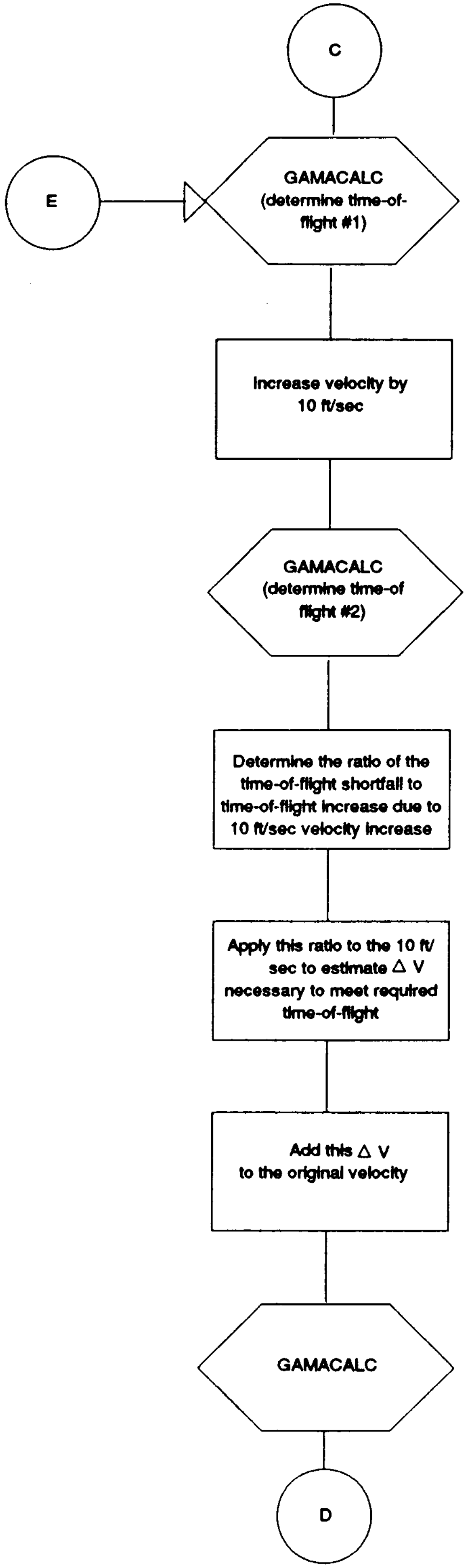

E
Have all 19 matrix rows been processed ?
NO
YES
Increment row counter by 1
Increment latitude
Reset column counter to 1
Reset longitude to initial input longitude
C
Set VELM to the SOI velocity associated with the minimum total Δ V
Set the SOI latitude and longitude to those associated with the minimum total Δ V
In-Program Subroutine
Print Report #2
Print Report #3
F

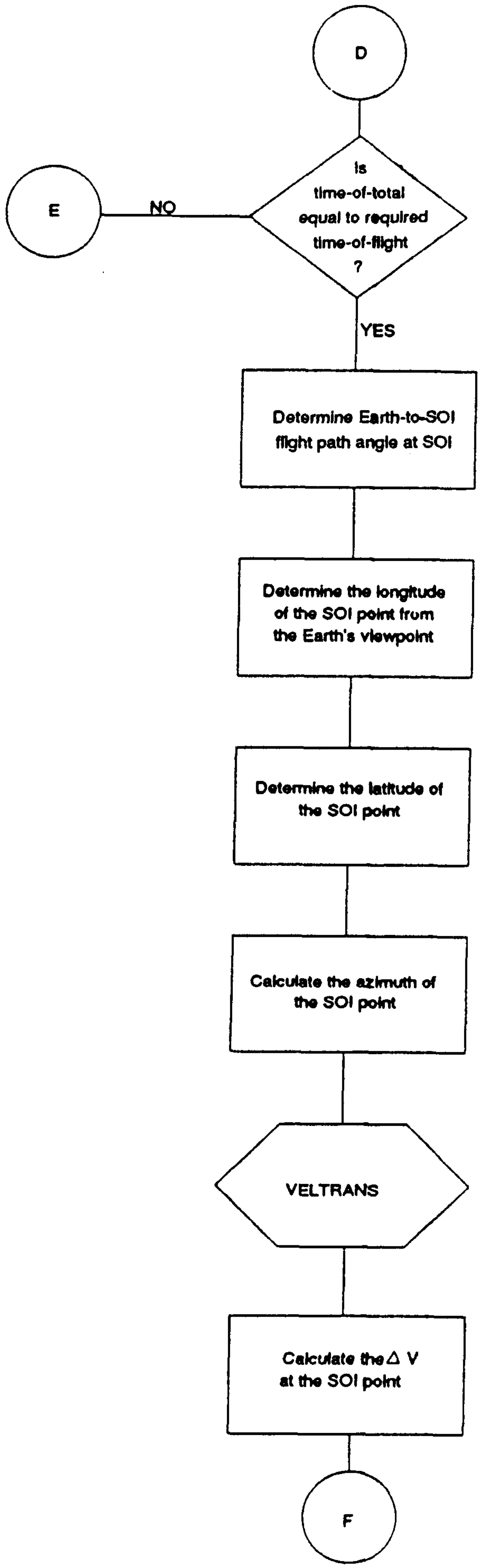

F
Print Report #4
Print Report #5
Print Report #6
END

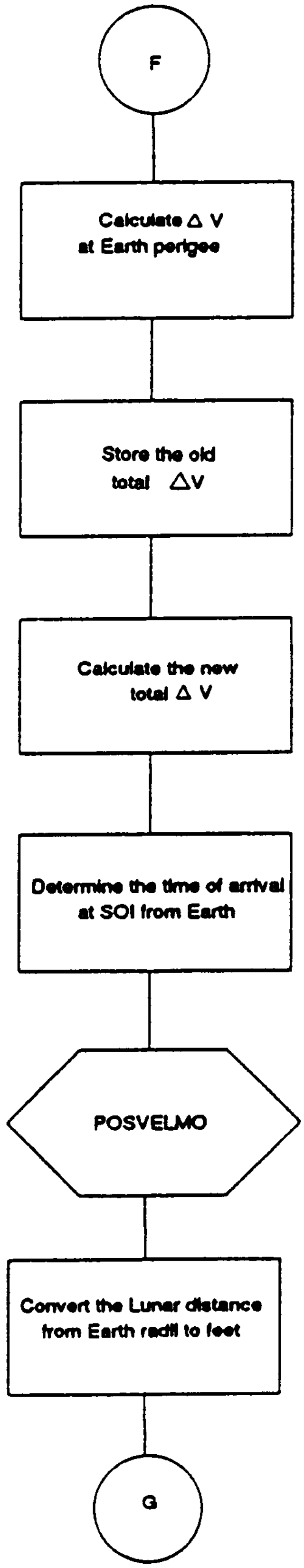

In-Program-Subroutine
H
Define variables for the sine and cosine of the SOI longitude and latitude
Define the angle between the Moon-to-Earth line and Moon-to-SOI line
Define variables for the sine and cosine of the Lunar orbit inclination
Calculate the ratio of Earth-Moon distance to Moon-SOI distance
Determine the distance form the moon to the SOI
Locate the SOI in rectangular coordinates centered at the Earth
Calculate the distance from the Earth to the SOI
A

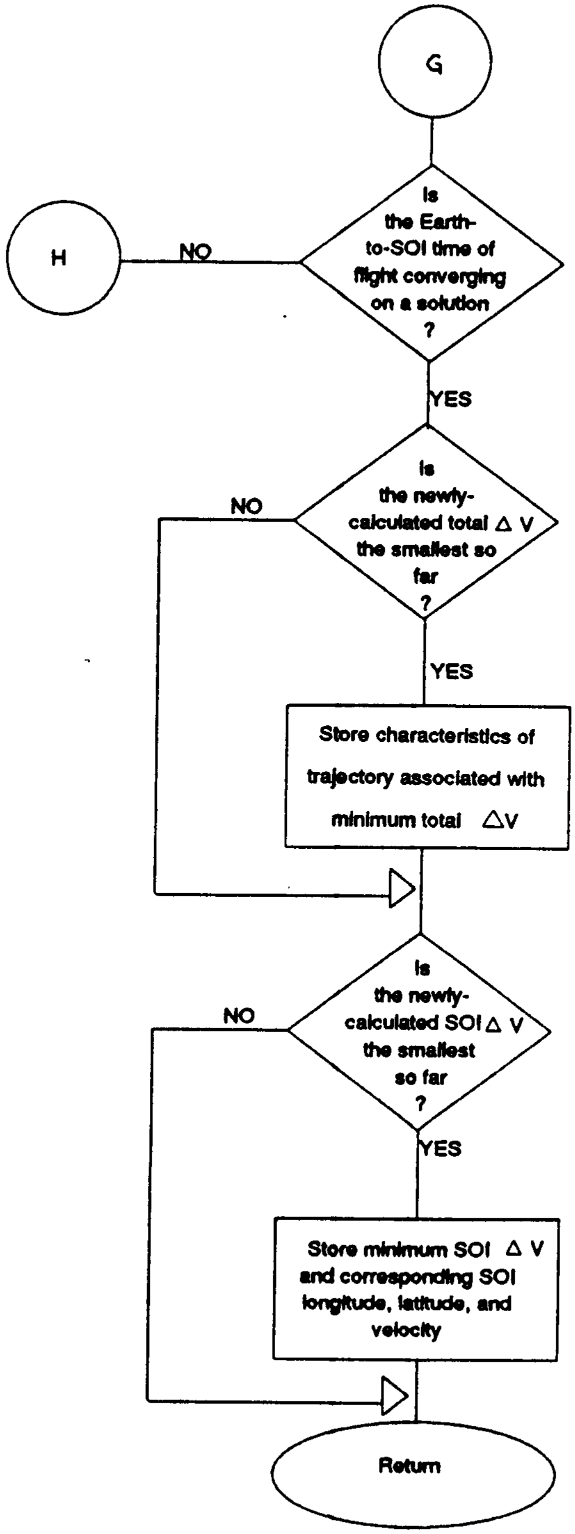

A
Identify the angle between the Earth-to-Moon line and Earth-toSOI line
Determine the Earth-based latitude and longitude of the SOI
Determine the Earth ascending node for the orbit in which the trans-SOI burn will occur
B

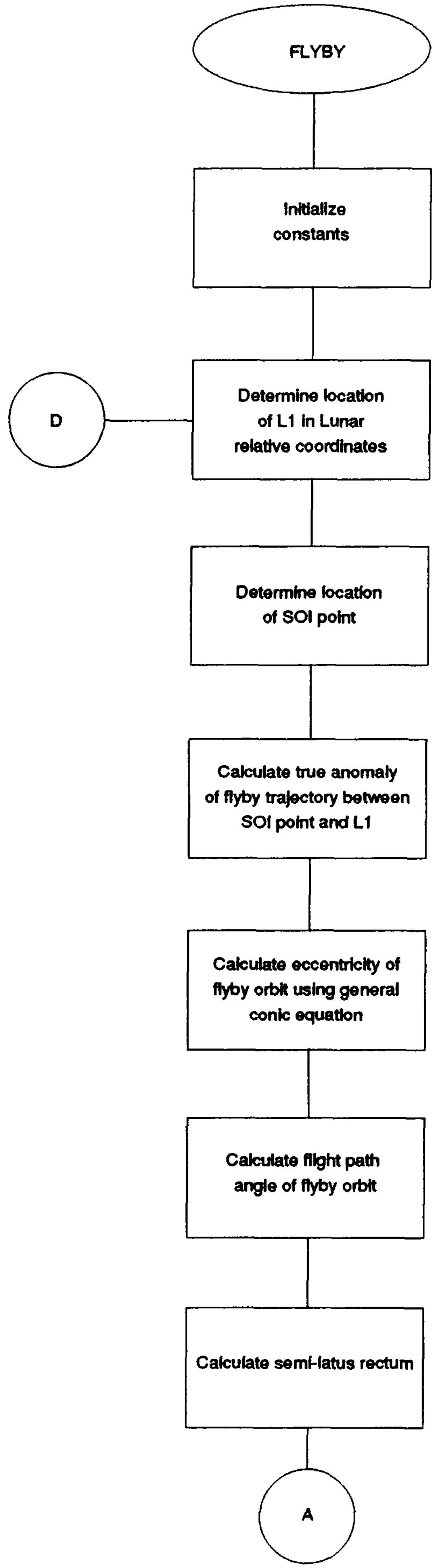

B
Determine the Lunar ascending node
Determine the rectangular coordinate components of the velocity vector at the SOI point for the flyby trajectory
Calculate the vis-viva parameter (Q) for the Earth-to-SOI trajectory
Calculate the minimum velocity required, such that apogee is just at SOI
Increase the velocity slightly, so that the calculations will converge
Is the most recently calculated SOI velocity (VELE)> the minimum velocity ?
NO
VELE = minimum velocity
YES
Temporarily store the Earth-to-SOI time of flight
C

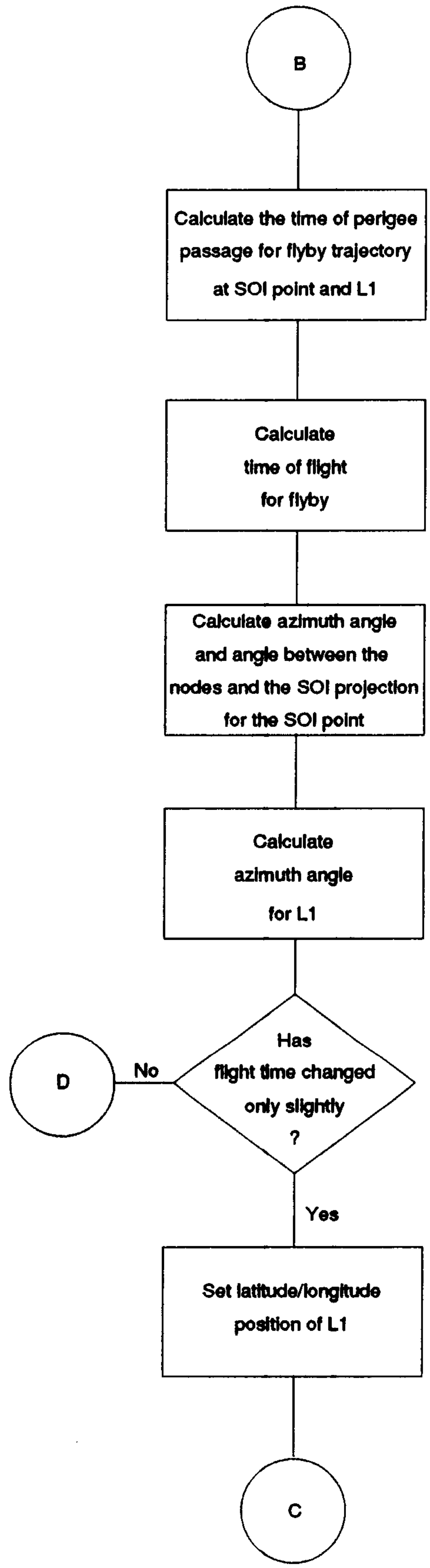

C
E
GAMACALC
(determine time-of-
flight #1)
Increase velocity by
10 ft/sec
GAMACALC
(determine time-of
flight #2)
Determine the ratio of the
time-of-flight shortfall to
time-of-flight increase due to
10 ft/sec velocity increase
Apply this ratio to the 10 ft/
sec to estimate Δ V
necessary to meet required
time-of-flight
Add this Δ V
to the original velocity
GAMACALC
D

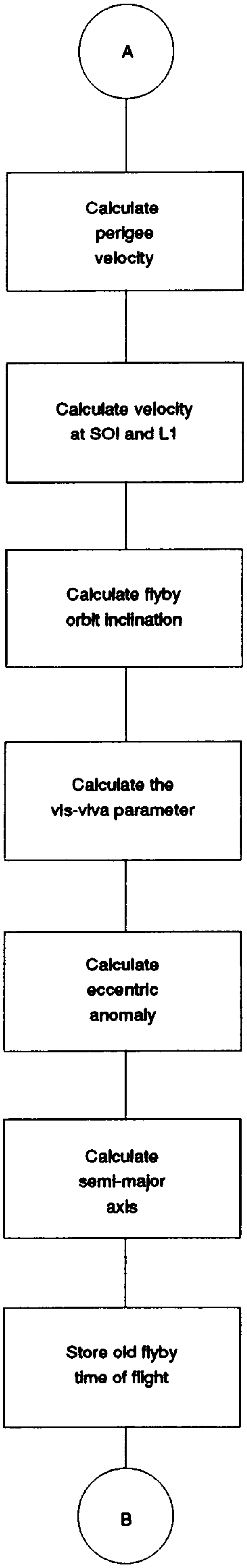

D
Is time-of-total equal to required time-of-flight ?
NO
E
YES
Determine Earth-to-SOI flight path angle at SOI
Determine the longitude of the SOI point from the Earth's viewpoint
Determine the latitude of the SOI point
Calculate the azimuth of the SOI point
VELTRANS
Calculate the Δ V at the SOI point
F

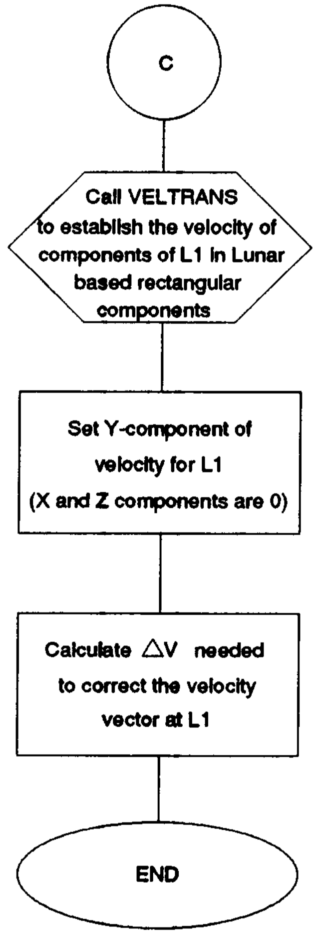

F
Calculate Δ V at Earth perigee
Store the old total ΔV
Calculate the new total Δ V
Determine the time of arrival at SOI from Earth
POSVELMO
Convert the Lunar distance from Earth radii to feet
G

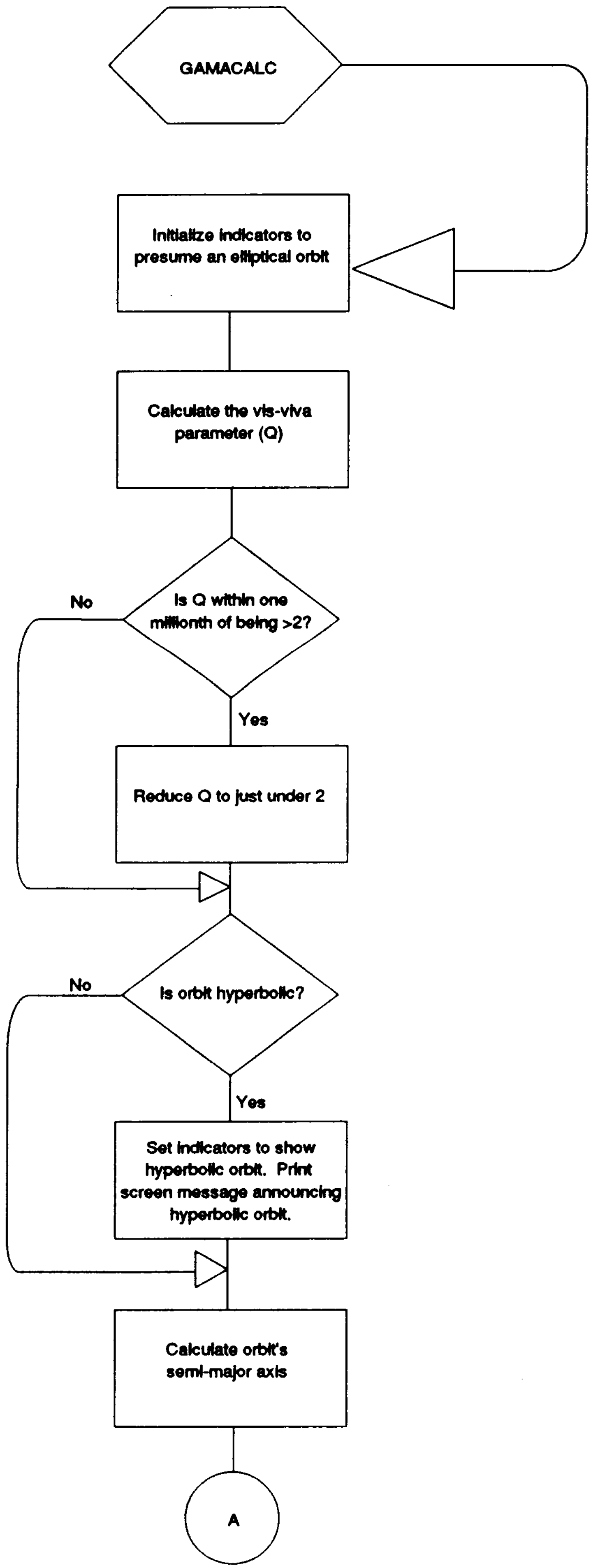

G
H
NO
Is the Earth-to-SOI time of flight converging on a solution ?
YES
NO
Is the newly-calculated total Δ V the smallest so far ?
YES
Store characteristics of trajectory associated with minimum total ΔV
NO
Is the newly-calculated SOI Δ V the smallest so far ?
YES
Store minimum SOI Δ V and corresponding SOI longitude, latitude, and velocity
Return

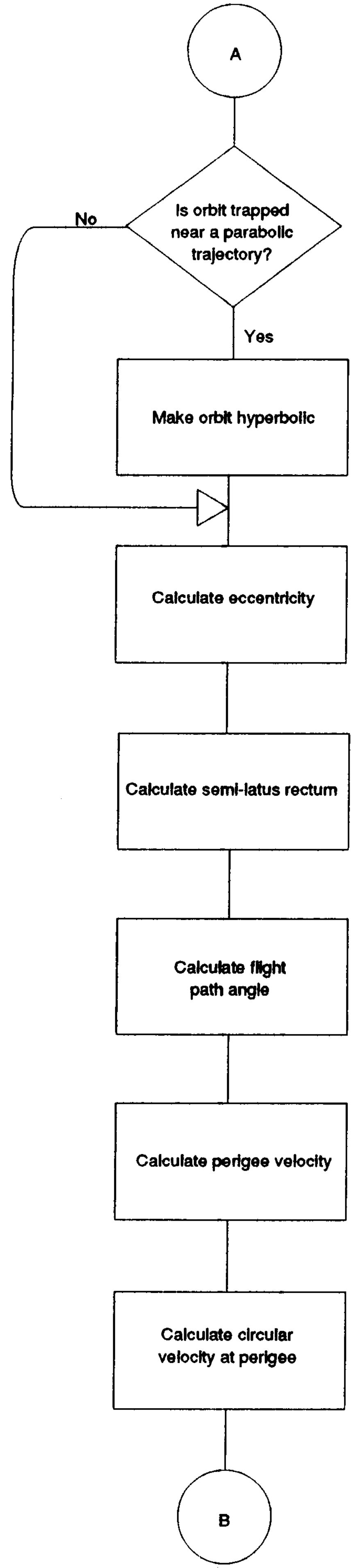

FLYBY
Initialize constants
D
Determine location of L1 in Lunar relative coordinates
Determine location of SOI point
Calculate true anomaly of flyby trajectory between SOI point and L1
Calculate eccentricity of flyby orbit using general conic equation
Calculate flight path angle of flyby orbit
Calculate semi-latus rectum
A

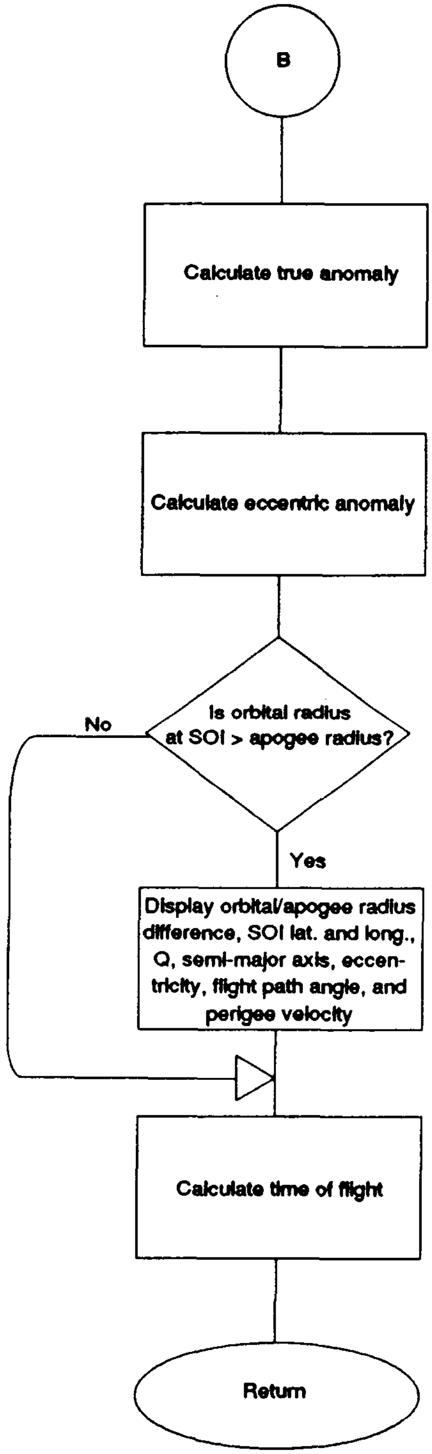

B
Calculate the time of perigee passage for flyby trajectory at SOI point and L1
Calculate time of flight for flyby
Calculate azimuth angle and angle between the nodes and the SOI projection for the SOI point
Calculate azimuth angle for L1
Has flight time changed only slightly ?
No
D
Yes
Set latitude/longitude position of L1
C

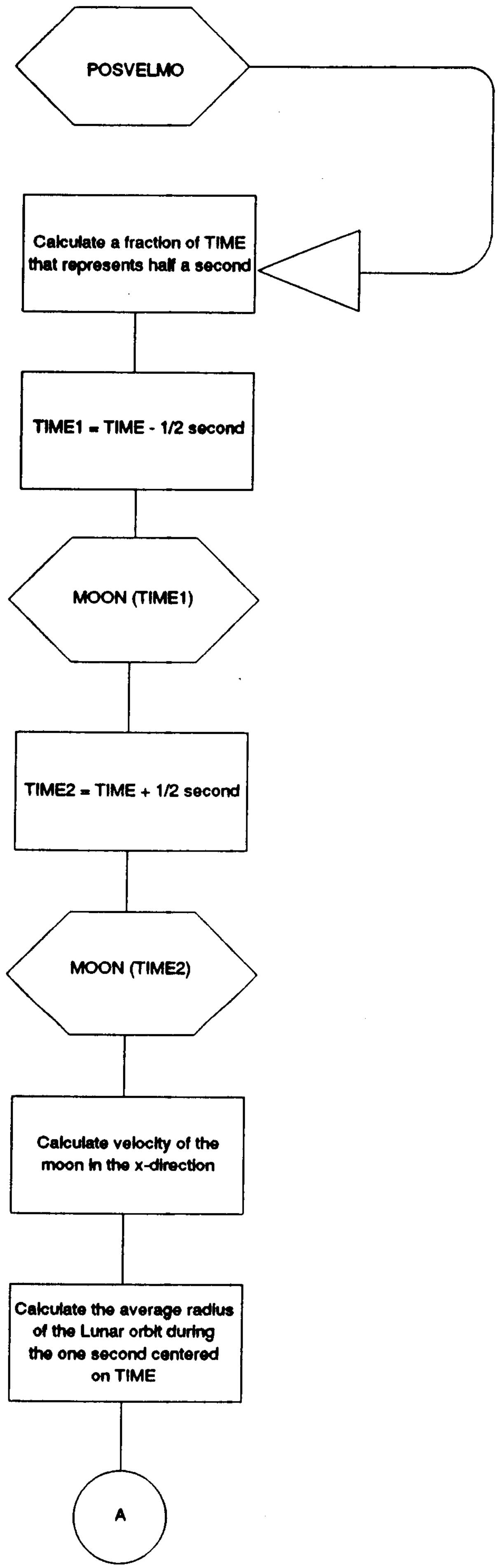

A
Calculate perigee velocity
Calculate velocity at SOI and L1
Calculate flyby orbit inclination
Calculate the vis-viva parameter
Calculate eccentric anomaly
Calculate semi-major axis
Store old flyby time of flight
B

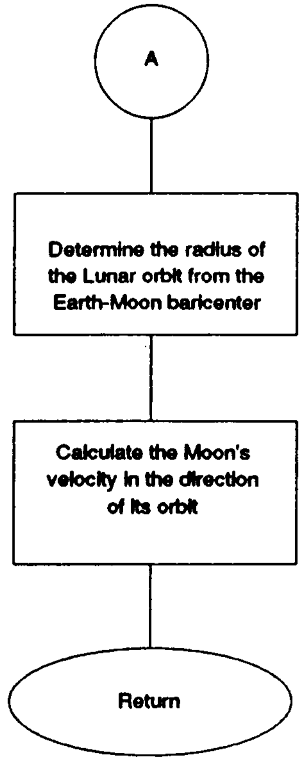

C
Call VELTRANS to establish the velocity of components of L1 in Lunar based rectangular components
Set Y-component of velocity for L1 (X and Z components are 0)
Calculate △V needed to correct the velocity vector at L1
END

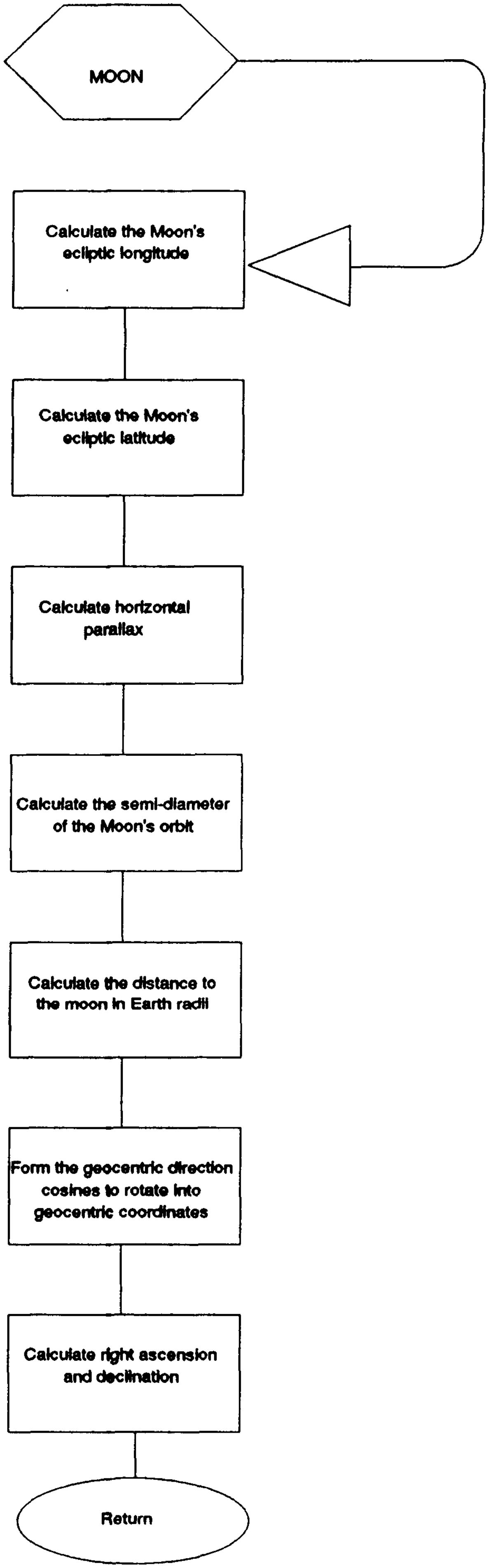

GAMACALC
Initialize indicators to presume an elliptical orbit
Calculate the vis-viva parameter (Q)
Is Q within one millionth of being >2?
No
Yes
Reduce Q to just under 2
Is orbit hyperbolic?
No
Yes
Set indicators to show hyperbolic orbit. Print screen message announcing hyperbolic orbit.
Calculate orbit's semi-major axis
A

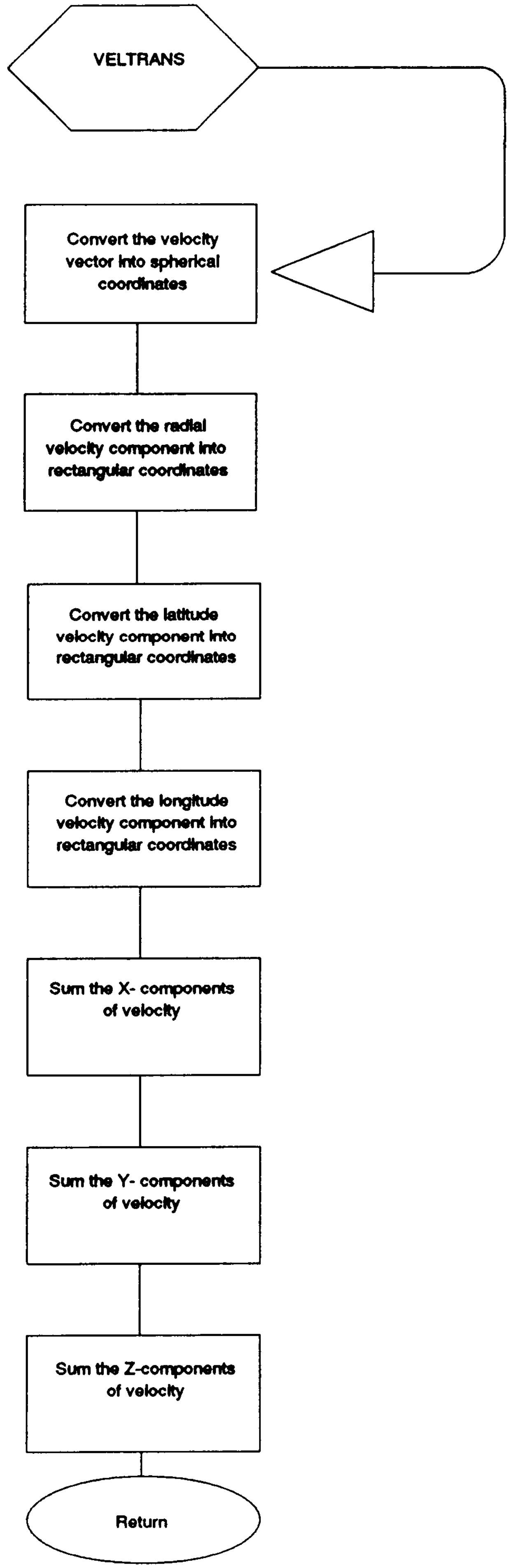

A
Is orbit trapped near a parabolic trajectory?
No
Yes
Make orbit hyperbolic
Calculate eccentricity
Calculate semi-latus rectum
Calculate flight path angle
Calculate perigee velocity
Calculate circular velocity at perigee
B


B
Calculate true anomaly
Calculate eccentric anomaly
Is orbital radius at SOI > apogee radius?
No
Yes
Display orbital/apogee radius difference, SOI lat. and long., Q, semi-major axis, eccentricity, flight path angle, and perigee velocity
Calculate time of flight
Return


POSVELMO
Calculate a fraction of TIME that represents half a second
TIME1 = TIME - 1/2 second
MOON (TIME1)
TIME2 = TIME + 1/2 second
MOON (TIME2)
Calculate velocity of the moon in the x-direction
Calculate the average radius of the Lunar orbit during the one second centered on TIME
A


A
Determine the radius of the Lunar orbit from the Earth-Moon baricenter
Calculate the Moon's velocity in the direction of its orbit
Return


MOON
Calculate the Moon's ecliptic longitude
Calculate the Moon's ecliptic latitude
Calculate horizontal parallax
Calculate the semi-diameter of the Moon's orbit
Calculate the distance to the moon in Earth radii
Form the geocentric direction cosines to rotate into geocentric coordinates
Calculate right ascension and declination
Return


VELTRANS
Convert the velocity vector into spherical coordinates
Convert the radial velocity component into rectangular coordinates
Convert the latitude velocity component into rectangular coordinates
Convert the longitude velocity component into rectangular coordinates
Sum the X- components of velocity
Sum the Y- components of velocity
Sum the Z-components of velocity
Return

Appendix B. Code Listing

```
C   ************************************************************
C   *** Libration Point Program (between L1 and Earth)       ***
C   ************************************************************
C   *   Written in Quick Basic by: Jack Funk                   *
C   *   Translated by Bill Engblom                             *
C   *   Documented by Bill Engblom                             *
C   ************************************************************
C
C       MAIN
C
      IMPLICIT REAL*16 (A-H,O-Z)
      CHARACTER*10 MD, TRAJ, TRAJM, TRAJE
      CHARACTER*8 TIMP
      CHARACTER*9 DATP
      CHARACTER*26 HEAD
      DIMENSION DELV(10,19), VELMOUT(10,19),ALONO(10),ALATP(19),
     * PAGE1(10), PAGE2(10,19), PAGE3(10,19),PAGE4(10,19),
     * PAGE5(10,19), PAGE6(10,19)
C
C     OPEN OUTPUT FILE
C
      OPEN (UNIT = 1,FILE = 'LP1.OUT' ,STATUS = 'NEW')
C     OUTPUT TO FILE; IP:  OUTPUT TO SCREEN; IS
      IP     = 1
      IS     = 5
      DPR    = 57.29578
      PI     = 3.1415926535
      CMUE   = 1.407647E+16
      CMUM   = 1.731432E+14
      FTNM   = 6076.115
      REE    = 20925741.
      REMO   = 5.7039E+6
      RREM   = 207559. * FTNM
      FTM    = .3048
      PRINT *, 'INPUT OUTBOUND OR RETURN '
      READ  (6, 5) MD
    5 FORMAT(A10)
      IF    (MD .EQ. 'RETURN' )THEN
        MOOD = -1
        HEAD = 'L1 TO EARTH FLYBY TRANSFER'
      ELSE
       MD = 'OUTBOUND'
       MOOD = 1
       HEAD = 'EARTH TO L1 FLYBY TRANSFER'
      ENDIF
C      PRINT *,'INPUT NODE OPTION 1 OR 2 '
C      READ  *, NP
   10 CONTINUE
      PRINT *, 'INPUT PERIGEE ALTITUDE OF EARTH ORBIT (NMI) '
```

```
      READ  *, HPE
      PRINT *, 'INPUT PERIGEE ALTITUDE OF LUNAR ORBIT (NMI) '
      READ  *, HPM
      RPE   =  HPE * FTNM + REE
      RPM   =  HPM * FTNM + REMO
      PRINT *, 'INPUT EARTH DEPARTURE JULIAN DATE '
      READ  *, TIMJ
      IF (TIMJ .EQ. 0.) TIMJ = 2451545.
      PRINT *, 'LONGITUDE FOR OUTBOUND TRAJECTORIES SHOULD BE
     * BETWEEN 0 AND -90 DEG'
      PRINT *, 'AND RETURN TRAJECTORIES BETWEEN 0 AND +90 DEG'
      PRINT *, 'INITIAL LONGITUDE'
      READ  *, ALONI
      PRINT *, 'INPUT INCREMENT FOR MAP '
      READ  *, DELLON
      PRINT *, 'INPUT INITIAL LATITUDE'
      READ  *, ALATI
      PRINT *, 'INPUT INCREMENT FOR MAP '
      READ  *, DELLAT
      PRINT *, 'INPUT EARTH ORBIT TO LUNAR ORBIT INCLINATION '
      READ  *, AINCEO
      AINCEO = AINCEO / DPR
      AINCE = AINCEO
      PRINT *, 'INPUT FLIGHT TIME  '
      READ  *, FTIM
      CALL DATE(DATP)
      CALL TIME(TIMP)
 15   CONTINUE
      WRITE (IP,7) HEAD, NP
 7    FORMAT (T12,' VELOCITY MAP FOR ',A26,' TRAJECTORIES
     * NODE OPTION  ',I1)
      WRITE (IP,17) DATP, TIMP
 17   FORMAT (T27,'DATE',A10,'   TIME',A10)
      WRITE (IP,27) TIMJ, HPE, HPM
 27   FORMAT (/,' JUILIAN DAY ', F8.0,'   PERIGEE ALT (NMI) EARTH = ',
     *F4.0, '  MOON = ', F6.0)
      WRITE (IP,37) FTIM, AINCEO * DPR
 37   FORMAT (' TRANSLUNAR FLIGHT TIME (HR) = ',F5.1,'  INCL
     *EARTH ' F5.1,'   INCL MOON PAGE3 ')
      IPRINT = 0
      II     = 1
      NN     = 1
      DVMIN  = 99999.
      YDLO   = QSQRT (CMUE / RREM)
      ALAT   = ALATI / DPR
      ALON   = ALONI / DPR
C     VELM   = VELMI
      DVSIM  = 99999.
 21   CONTINUE
```

```
C       VEL            = VELM
        DELV(NN,II)    = 99999.
        PAGE1(NN)    = 0.0
        PAGE2(NN,II) = 0.0
        PAGE3(NN,II) = 0.0
        PAGE4(NN,II) = 0.0
        PAGE5(NN,II) = 0.0
        PAGE6(NN,II) = 0.0
C        PRINT *,' '
C        PRINT *,'  ALAT    ALON    VEL    DVT      DVSI    MOON   EARTH

C       *TIMEE  TIMEM  RREM'
 22     CONTINUE
C
C       CALCULATE FLYBY TRAJECTORY CHARACTERISTICS
        CALL FLYBY(ALON,ALAT,RREM,RPM,GAMAM,VELM,VPM,AINCM,DELVLP1,
       *AZMM,AVM,TIMM,MOOD)
            ICALL = 1
            GOTO 25
 1000       CONTINUE
C            WRITE (IS,47) ALAT*DPR,ALON*DPR,VELM,DVTOTAL,DELVEL,
C       *    TRAJM$,TRAJE$,TIEM,TIMM,RREMER,PHIE*DPR
C   47       FORMAT (1X,F5.0,2X,F5.0,2X,F6.0,2X,F7.0,2X,F6.0,2X,
C       *    A10,2X,A10,2X,F6.1,2X,F6.1,2X,F7.3,2X,F5.0)
            IF (QABS(TIEM) .LE. .0000000000000001) GOTO 23
C           STORE VALUES FOR OUTPUT
            IF(DVTOTAL .LT. DELV(NN,II) ) THEN
              DELV (NN,II)     = DVTOTAL
              PAGE1 (NN)       = DVTOTAL
              PAGE2 (NN,II)    = DELVEL
              VELMOUT (NN,II) = VELM
              ALATP (II)       = ALAT   * DPR
              PAGE3 (NN,II)    = AINCM  * DPR
              PAGE4 (NN,II)    = DELVLP1
              PAGE5 (NN,II)    = DVCIRE
              PAGE6 (NN,II)    = AZMM * DPR
C             WRITE (IS,57) ALAT * DPR, ALON * DPR, VELMOUT (NN,II)
C       *      ,DELV(NN,II), PAGE2(NN,II), TRAJM, TRAJE, TIEM,
C       *      TIMM, RREMER
C  57          FORMAT (1X,F6.1,1X,F6.1,2X,F7.0,1X,F8.1,1X,F7.1,1X,
C       *              A5,1X,A5,1X,F6.1,1X,F6.1,1X,F8.3)
C              WRITE (IS,67) VXE, VXM, VYE, VYM, VZE, VZM
C  67          FORMAT (' VXE ',F7.1,' VXM ',F7.1,' VYE ',F7.1,' VYM ',
C       *                F7.1, ' VZE ', F7.1, 'VZM ',F7.1)
            ENDIF
 23      CONTINUE
        ALONO(NN) = ALON * DPR
        IF (NN .EQ. 10 .AND. IPRINT .EQ. 0 ) THEN
            WRITE  (IP,77)  ALONO(1), ALONO(2), ALONO(3), ALONO(4),
```

```
     *    ALONO(5), ALONO(6), ALONO(7),ALONO(8),ALONO(9),ALONO(10)
77        FORMAT (/,' ALON> ', 10(2X,F5.1 ))
          WRITE (IP,87)
87        FORMAT (' ALAT')
      ENDIF
      IF (NN .LT. 10) THEN
        ALON = (ALONI + DELLON * QFLOAT (NN ) ) / DPR
        NN   = NN + 1
        GOTO 21
      ENDIF
      IPRINT = 1
      WRITE (IP,97) ALAT * DPR, PAGE1(1),PAGE1(2),
     *PAGE1(3),PAGE1(4),PAGE1(5),PAGE1(6),
     *PAGE1(7),PAGE1(8),PAGE1(9),PAGE1(10)
 97   FORMAT (1X, F5.1, 2X, 10(1X,F6.0))
      IF (II .LT. 19) THEN
        ALAT = (ALATI + DELLAT * QFLOAT (II ) ) / DPR
        II   = II+1
        NN   = 1
        ALON = ALONI / DPR
        GOTO 21
      ENDIF
      VELM  = VELMMIN
      ALAT  = ALATMIN
      ALON  = ALONMIN
      ICALL = 2
      GOTO 25
2000 CONTINUE
      WRITE  (IP,107) NP
107   FORMAT (///,T23,'MINIMUM VELOCITY PRINT NODE OPTION ',I1)
      WRITE  (IP,117) DATP, TIMP
117   FORMAT (T27,'DATE',A10,' TIME',A10)
      WRITE  (IP,127) FTIM, ALATMIN * DPR, ALONMIN * DPR, AINCMP * DPR,
     * AINCEO * DPR
127   FORMAT (/,' FLIGHT TIME = ',F4.0,' LAT = ',F6.1,' LON = ',F6.1,
     *         ' LUNAR AINC = ', F5.1,' EARTH AINC = ',F5.1)
      WRITE  (IP,137)
137   FORMAT (' BODY    VX     VY      VZ      VEL    GAMA    AZM
     *AINC   ANODE  TIME  DVCIR')
      WRITE  (IP,147) VXMP,VYMP,VZMP,VMAGMP,GAMAMP * DPR,
     *AZMMP * DPR, AINCMP * DPR, ANODEMP*DPR,TIMMP,DELVLP1P,DELVELP
147   FORMAT (' MOON ',4(1X,F6.0),1X,F5.1,1X,F6.1,2X,F5.1,2X,F6.1,
     * 1X,F5.1,1X,F7.0,' DVPHER = ',F7.0)
      WRITE  (IP,1475) VXEP, VYEP, VZEP, VELEP,GAMAEP*DPR,AZMEP*DPR,
     *             AINCEP * DPR,ANODEEP*DPR,TIEMP,DVCIREP,DVTOTALP
1475 FORMAT (' EARTH',4(1X,F6.0),1X,F5.1,1X,F6.1,2X,F5.1,2X,F6.1,
     * 1X,F5.1,1X,F7.0,' DVTOTAL = ', F7.0)
      WRITE (IP,157) CHAR(12)
157   FORMAT (' ',A1)
```

```
      WRITE  (IP,167) HEAD,NP
167   FORMAT (T10,'MAP OF DELTA VELOCITY AT SPHERE OF INFLUENCE
     *FOR ',A26,' TRAJECTORIES NODE OPTION ',I1)
      WRITE  (IP,177) DATP,TIMP
177   FORMAT (T27,'DATE',A10,'  TIME',A10)
      WRITE  (IP,187)  HEAD,FTIM, AINCEO * DPR, AINCM * DPR
187   FORMAT ('  ',A26,' FLIGHT TIME (HR) = ',F6.1,' INCL EARTH ',
     *F6.1,'  INCL MOON = ',F5.1)
      WRITE  (IP,197) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
     *ALONO(5),ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
197   FORMAT (/,' ALON> ',10(2X, F5.1))
18    CONTINUE
      WRITE  (IP,207)
207   FORMAT (' ALAT')
      DO 28 NPI = 1 , 19
        WRITE (IP,217) ALATP(NPI), PAGE2(1,NPI),PAGE2(2,NPI),
     *  PAGE2(3,NPI),PAGE2(4,NPI),PAGE2(5,NPI),PAGE2(6,NPI),
     *  PAGE2(7,NPI),PAGE2(8,NPI),PAGE2(9,NPI),PAGE2(10,NPI)
217     FORMAT(1X,F5.1,1X,10(1X,F6.1))
28    CONTINUE
      WRITE  (IP, 227 ) CHAR(12)
227   FORMAT (' ', A1)
      WRITE  (IP, 237 ) HEAD , NP
237   FORMAT (T6,'MAP OF INCLINATION OF FLY-BY ORBIT FOR ',A26,
     * ' TRAJECTORIES NODE OPTION ',I1)
      WRITE  (IP, 247 ) DATP, TIMP
247   FORMAT (T27,'DATE',A10,'   TIME',A10)
      WRITE  (IP, 257 ) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
257   FORMAT ('  ',A26,'  FLIGHT TIME (HR) = ',F6.1,'  INCL EARTH ',
     *F6.1,'  INCL MOON = ',F6.1)
      WRITE  (IP,267) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
     *ALONO(5),ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
267   FORMAT (/,' ALON> ', 10(2X, F5.1))
      WRITE (IP,277)
277   FORMAT (' ALAT')
      DO 48 NPI = 1 , 19
        WRITE (IP,287) ALATP(NPI),PAGE3(1,NPI),PAGE3(2,NPI),
     *  PAGE3(3,NPI),PAGE3(4,NPI),PAGE3(5,NPI),PAGE3(6,NPI),
     *  PAGE3(7,NPI), PAGE3(8,NPI),PAGE3(9,NPI),PAGE3(10,NPI)
287     FORMAT (' ',F5.1,1X,10(2X,F5.1))
48    CONTINUE
      WRITE  (IP, 297) CHAR(12)
297   FORMAT (' ',A1)
      WRITE  (IP, 307 ) HEAD,NP
307   FORMAT (T6,'MAP OF DELTA VELOCITY AT L1
     * FOR ',A26,' TRAJECTORIES NODE OPTION ',I1)
      WRITE  (IP, 317 ) DATP,TIMP
317   FORMAT (T27,'DATE',A10,'   TIME',A10)
      WRITE  (IP, 327 ) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
```

```
327  FORMAT (1X, A26,'  FLIGHT TIME (HR) = ',F5.1,'   INCL EARTH ',
    *                 F5.1,'  INCL MOON = ',F5.1)
     WRITE  (IP, 337) ALONO(1),ALONO(2),ALONO(3),ALONO(4),ALONO(5),
    *ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
337  FORMAT (/,' ALON> ', 10(2X, F5.1))
     WRITE (IP,347)
347  FORMAT ('  ALAT')
     DO 58 NPI = 1 , 19
       WRITE  (IP,357) ALATP(NPI),PAGE4(1,NPI),PAGE4(2,NPI),
    *  PAGE4(3,NPI),PAGE4(4,NPI),PAGE4(5,NPI),PAGE4(6,NPI),
    *  PAGE4(7,NPI),PAGE4(8,NPI),PAGE4(9,NPI),PAGE4(10,NPI)
357    FORMAT(1X,F5.1,2X,10(1X,F6.0))
58   CONTINUE
     WRITE(IP,367) CHAR(12)
367  FORMAT(' ',A1)
     WRITE(IP,377) HEAD, NP
377  FORMAT(T2,'DELTA VELOCITY AT EARTH FOR ',A26,' TRAJECTORY
    * NODE OPTION ',I1)
     WRITE(IP,387) DATP, TIMP
387  FORMAT(T27,'DATE ', A10,'   TIME ', A10)
     WRITE(IP,397) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
397  FORMAT('   ',A26,'  FLIGHT TIME (HR) = ',F6.1,'  INCL EARTH ',
    *F6.1,'  INCL MOON = ',F6.1)
     WRITE(IP,407) ALONO(1), ALONO(2), ALONO(3), ALONO(4),
    *ALONO(5), ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
407  FORMAT(/,' ALON> ', 10(2X, F5.1))
     WRITE(IP,417)
417  FORMAT(' ALAT')
     DO 68 NPI = 1, 19
       WRITE(IP,427) ALATP(NPI), PAGE5(1,NPI), PAGE5(2,NPI),
    *  PAGE5(3,NPI), PAGE5(4,NPI), PAGE5(5,NPI), PAGE5(6,NPI),
    *  PAGE5(7,NPI), PAGE5(8,NPI), PAGE5(9,NPI), PAGE5(10,NPI)
427  FORMAT (1X,F5.1,2X,10(1X,F6.0))
68   CONTINUE
     WRITE(IP,437) CHAR(12)
437  FORMAT(' ',A1)
     WRITE(IP,447) HEAD, NP
447  FORMAT(T2,'MAP OF AZMM FOR ',A26,'TRAJECTORY
    * NODE OPTION ',I1)
     WRITE(IP,457) DATP, TIMP
457  FORMAT(T27,'DATE',A10,'   TIME',A10)
     WRITE(IP,467) HEAD, FTIM, AINCEO * DPR, AINCM * DPR
467  FORMAT(1X,A26,'  FLIGHT TIME (HR) = ',F5.1,'  INCL EARTH ',
    *F5.1,'  INCL MOON = ',F5.1)
     WRITE(IP,477) ALONO(1), ALONO(2), ALONO(3), ALONO(4), ALONO(5),
    * ALONO(6), ALONO(7), ALONO(8), ALONO(9), ALONO(10)
477  FORMAT(/,' ALON> ', 10(2X, F5.1))
     WRITE(IP,487)
487  FORMAT(' ALAT')
```

```
      DO 78 NPI = 1, 19
        WRITE(IP,497) ALATP(NPI), PAGE6(1,NPI), PAGE6(2,NPI),
     *  PAGE6(3,NPI), PAGE6(4,NPI), PAGE6(5,NPI), PAGE6(6,NPI),
     *  PAGE6(7,NPI), PAGE6(8,NPI), PAGE6(9,NPI), PAGE6(10,NPI)
 497    FORMAT(1X,F5.1,2X,10(1X,F6.0))
 78   CONTINUE
      GOTO 3000
 25   CONTINUE
      COSALAT = QCOS (ALAT )
      SINALAT = QSIN (ALAT )
      COSALON = QCOS (ALON )
      SINALON = QSIN (ALON )
      COSPHI  = COSALAT * COSALON
      COSAINC = QCOS (AINCM / DPR )
      SINAINC = QSIN (AINCM / DPR )
      RMR     = COSPHI +QSQRT (COSPHI ** 2. - (1. - CMUE / CMUM ))
      RRM     = RREM * 0.15
C     CALCULATION OF XYZ COORDINATES RELATIVE TO EARTH AT SPHERE
C     OF INFLUENCE
C     SPHERE OF INFLUENCE HAS BEEN REDEFINED EQUAL TO THE LIBRATION
C     POINT #1 RADIUS
      XXM     = RRM  * COSALAT * COSALON
      XX      = -XXM + RREM
      YY      = -RRM * COSALAT * SINALON
      ZZ      =  RRM * SINALAT
      YYM     = -YY
C      WRITE (IS,507) XX,YY,ZZ
C 507  FORMAT(' XYZ POSITION AT SPHERE ',F11.0,1X,F11.0,1X,F11.0)
      RRE     = QSQRT(XX**2.+YY**2.+ZZ**2.)
      COSANGA = XX / RRE
      SINANGA = QSQRT (YY ** 2. + ZZ ** 2. ) / RRE
C      ANGA    = QATAN (SINANGA / COSANGA )
      ALONX   = QATAN (YY / XX )
      ALATX   = QATAN (ZZ / QSQRT (XX ** 2. + YY **2. ) )
      ANODEE  = ALONX - QASIN (QTAN (-ALATX ) / QTAN (AINCE ) )
C     END SPHERE OF INFLUENCE CALC
  30  CONTINUE
      ANODEM  = ALON-AVM
      IF(ANODEM .LT.0.)  ANODEM = ANODEM + 2.*PI
      CALL VELTRANS(VELM,GAMAM,AZMM,ALAT,ALON,VXM,VYM,VZM,VMAGM,DPR)
      VXM = VXM+XDLO
      VYM = VYM+YDLO
      QQEMIN = 2.1*RPE/(RRE+RPE)
      VELEMIN = QSQRT(QQEMIN*CMUE/RRE)
      IF(VELE .LT. VELEMIN) VELE = VELEMIN
      TIEMS = TIEM
C     TIMEES CHANGED TO TIEMS, TIMEE CHANGED TO TIEM
  35  CONTINUE
      CALL GAMACALC(RPE,VELE,RRE,CMUE,COSGAME,VPE,VCIRE,
```

```
     *TIME1,TRAJE,DPR, ALAT,ALON,FTNM)
      VELE2 = VELE + 10.
      CALL GAMACALC(RPE,VELE2,RRE,CMUE,COSGAME,VPE,VCIRE,
     *TIME2,TRAJE,DPR,ALAT,ALON,FTNM)
      DELVELE = 10./(TIME2-TIME1)*(FTIM-TIME1-TIMM)
      IF(DELVELE .LT. 500.) THEN
        VELE =VELE + DELVELE
      ELSE
        VELE = VELE + DELVELE/QABS(DELVELE)*500.
      ENDIF
      CALL GAMACALC(RPE,VELE,RRE,CMUE,COSGAME,VPE,VCIRE,TIEM,
     *TRAJE,DPR,ALAT,ALON,FTNM)
      IF(TIEM .EQ. 0.) GOTO 60
      TIMT = TIEM+TIMM
      IF(ABS(FTIM-TIMT) .GT. 1.) GOTO 35
      GAMAE = QFLOAT(MOOD)*QATAN(QSQRT(1.-COSGAME**2.)/COSGAME)
      ALONE = 180. / DPR + ALONX
      ALATE = QATAN(ZZ/QSQRT(XX**2.+YY**2.))
   50 CONTINUE
      AINCE = AINCEO
C     'CALCULATION OF EARTH ORBIT AZIMUTH AT SPHERE OF INFLUENCE
      IF(AINCE .NE. 0.0) THEN
        PHIE = PI-QASIN(ZZ/(RRE*QSIN(AINCE)))
        AZME = QATAN2(1./QTAN(AINCE),QCOS(PHIE))
      ELSE
        AZME = PI/2
      END IF
C
      CALL VELTRANS(VELE,GAMAE,AZME,ALATE,ALONE,VXE,VYE,VZE,VELE,
     * DPR)
      DELVEL = QSQRT((VXE-VXM)**2.+(VYE-VYM)**2.+(VZE-VZM)**2.)
      DVCIRE = VPE - VCIRE
      DVE = QSQRT (VCIRE ** 2. + VPE**2.-2. * VCIRE*VPE*
     *COS (AINCEO - AINCE ) )
      DVTSAV  = DVTOTAL
      DVTOTAL = DELVEL + DELVLP1 + DVCIRE
C       CALCULATE POSITION AND VELOCITY OF MOON
C       ITTERATE FOR FLIGHT TIM TO SPHERE OF INFLUENCE
      TIM = (TIMJ-2451545.)/36525.+TIEM/876600.
      CALL POSVELMO(TIM,RREMER,XDLO,YDLO)
      RREM = RREMER*20295741.
      IF(ABS(TIEM-TIEMS) .GT. .1) GOTO 25
      IF(DVTOTAL.LT. DVMIN)THEN
        DVMIN = DVTOTAL
        ALONMIN = ALON
        ALATMIN = ALAT
        VELMMIN=VELM
        VXMP = VXM
        VYMP = VYM
```

```
        VZMP = VZM
        VMAGMP = VMAGM
        GAMAMP = GAMAM
        AZMMP = AZMM
        AINCMP = AINCM
        ANODEMP = ANODEM
        TIMMP = TIMM
        DELVLP1P = DELVLP1
        DELVELP = DELVEL
        VXEP = VXE
        VYEP = VYE
        VZEP = VZE
        VELEP = VELE
        GAMAEP = GAMAE
        AZMEP = AZME
        AINCEP = AINCE
        ANODEEP = ANODEE
        TIEMP = TIEM
        DVCIREP = DVCIRE
        DVTOTALP = DVTOTAL
      ENDIF
      IF(DELVEL .LT. DVSIM)THEN
        DVSIM = DELVEL
        ALONSIM = ALON
        ALATSIM = ALAT
        VELMSIM = VELM
        OMEGE = PHIE - THETAE
      ENDIF
60    CONTINUE
      WRITE (IS,517) ALON*DPR, ALAT*DPR
517   FORMAT ('  ALON = ',F6.1,'  ALAT = ',F6.1)
      IF (ICALL.EQ.1) GOTO 1000
      IF (ICALL.EQ.2) GOTO 2000
3000  STOP
      END

      SUBROUTINE FLYBY(ALON,ALAT,RREM,RPM,GAMAM,VELM,VPM,AIM,
     *DELVLP1,AZMM,AVM,TIEMM,MOOD)
      IMPLICIT REAL * 16 (A-H,O-Z)
      IS = 5
      DPR = 57.29578
      PI = 3.141593
      CMUE = 1.407647E+16
      CMUM = 1.731400E+14
      FTNM = 6076.115
      REE = 20925741.
      REMO = 5.7039E+6
      OMEGM = .9582117947E-2
      FTM = .3048
```

```
      R1 = .15 * RREM
C      PRINT *,' TIMEM  INCL  THETAM  GAMAM   VELM      QQ   EEM'
   10 CONTINUE
C     LOCATION OF LP#1 IN LUNAR RELATIVE COORDINATES.  LIBRATION
C     POINT ROTATES WITH THE MOON AND THEREFORE IS EFFECTED BY
C     FLYBY TIME
      X1 = -R1 * QCOS(OMEGM*TIEMM)
      Y1 = -R1 * QSIN(OMEGM*TIEMM) * FLOAT(MOOD)
      Z1 = 0.
C     R1 IS LP#1, R2 IS SPACECRAFT AT SPHERE OF INFLUENCE. SPHERE
C     OF INFLUENCE IS DEFINED EQUAL TO RADIUS OF LP#1.  THIS IS
C     SLIGHTLY LARGER THAN THE ONE USED IN THE PLANE CHANGE PROGRAM.
C     HOWEVER, THIS CHANGE SIMPLIFIES THE CALCULATION OF THE FLYBY
C     TRAJECTORY
      X2 = R1 * QCOS(ALAT) * QCOS(ALON)
      Y2 = R1 * QCOS(ALAT) * QSIN(ALON)
      Z2 = R1 * QSIN(ALAT)
C     CALCULATION OF TRUE ANOMALY (THETA)
      DOT = (X1*X2 + Y1*Y2 + Z1*Z2)
      COS2THETA = DOT/R1/R1
      THETA = PI - QACOS(COS2THETA)/2.
      COSTHETAM = QCOS(THETA)
C     CALCULATION OF ECCENTRICITY OF FLYBY ORBIT FROM RP, R1, AND
C     THETA
      EEM = (RPM/R1 - 1.)/(COSTHETAM - RPM/R1)
      COSGAMA = (1.+EEM*COSTHETAM)/QSQRT(1.+2.*EEM*COSTHETAM+EEM*EEM)
C     CALCULATE FLIGHT PATH ANGLE.  GAMA IS POSITIVE AT L1 AND NEGA-
C      TIVE AT SOI FOR OUTBOUND AND REVERSED FOR INBOUND TRAJECTORIES

      GAMAM = QACOS(COSGAMA) * FLOAT(MOOD)
      PPM = RPM * (1. + EEM)
      VPM = QSQRT(CMUM * (2./RPM + (EEM*EEM - 1.)/PPM))
      VELM = QSQRT(CMUM * (2./R1 + (EEM*EEM - 1.)/PPM))
C     R1 CROSS R2 TO CALCULATE INCLINATION FROM Z3
      X3 = Y1*Z2 - Z1*Y2
      Y3 = Z1*X2 - X1*Z2
      Z3 = X1*Y2 - Y1*X2
      AMAG = QSQRT(X3*X3 + Y3*Y3 + Z3*Z3)
      AIM = QACOS(Z3/AMAG)
      QQM = R1 * VELM*VELM/CMUM
      COSAEX = (EEM + COSTHETAM)/(1. + EEM * COSTHETAM)
      AAX = R1/(2. - QQM)
      TIEMMS = TIEMM
      IF (QQM .GT. 2.) GOTO 101
C     CALCULATE FLIGHT TIME FOR ELLIPTIC ORBITS
      IF (ERRRP .GT. 0.) THEN
        TIMX = 0.
        GOTO 199
      ENDIF
```

```
      AEX = QACOS(COSAEX)
      SINAEX = QSQRT(1. - COSAEX*COSAEX)
      TIMX = AAX*QSQRT(AAX/CMUM)*(AEX-EEM*SINAEX)/3600.
      GOTO 199
  101 CONTINUE
C     CALCULATE FLIGHT TIME FOR HYPERBOLIC ORBITS
      COSHF = COSAEX
      SINHF = QSQRT(COSHF*COSHF - 1.)
      FFX = QLOG(COSHF + QSQRT(COSHF*COSHF - 1.))
      TIMX =  -AAX*QSQRT(-AAX/CMUM)*(EEM*SINHF - FFX)/3600.
  199 CONTINUE
      TIEMM = TIMX * 2.
C     CALCULATION OF AZIMUTH AND ANGLE AV AT R2
      ANODE = OMEGM * TIEMM * FLOAT(MOOD)
      IF (ALAT*FLOAT(MOOD) .GE. 0.) ANODE = ANODE + PI
      AVM = ALON - ANODE
      COSPHIM = QCOS(ALAT) * (QCOS(ANODE) * QCOS(ALON) +
     *  QSIN(ANODE) * QSIN(ALON))
      APHIM = QACOS(COSPHIM)
      IF (QABS(ALAT) .GT. 89.9/DPR) THEN
        AZMM = (PI + PI * ALAT/QABS(ALAT))/2.
        GOTO 200
      ENDIF
      SINAZM = QCOS(AIM)/QCOS(ALAT)
      COSAZM = QSIN(AIM) * QCOS(APHIM)/QCOS(ALAT)
      AZMM = QATAN2(SINAZM, COSAZM)
  200 CONTINUE
C     CALCULATE AZIMUTH FOR L1
      ALATL1 = 0.
      ALONL1 = QATAN2(Y1, X1)
      COSPHIL1 = QCOS(ALATL1) * (QCOS(ANODE) * QCOS(ALONL1) +
     * QSIN(ANODE) * QSIN(ALONL1))
      APHIL1 = QACOS(COSPHIL1)
      SINAZM = QCOS(AIM)/QCOS(ALATL1)
      COSAZM = QSIN(AIM) * QCOS(APHIL1)/QCOS(ALATL1)
      AZMLP1 = QATAN2(SINAZM, COSAZM)
C     PRINT DATA AND ITERATE FOR FLIGHT TIME
      IF (QABS(TIEMM - TIEMMS) .GT. 0.1) GOTO 10
C      WRITE (IS,17) TIEMM, AIM*DPR, THETA*DPR, GAMAM*DPR,
C     *VELM, QQM, EEM
C   17 FORMAT (1X,F5.1,3(2X,F5.1),2X,F10.2,2X,F10.4,2X,F10.6)
      IF (QABS(TIEMM - TIEMMS) .GT. 0.1) GOTO 10
C     LOCATION OF LB#1 IN LUNAR RELATIVE COORDINATES
      ALATLP1 = 0.
      ALONLP1 = 180.
      CALL VELTRANS(VELM,GAMAM,AZMLP1,ALATLP1,ALONLP1,VXLP1,
     *VYLP1,VZLP1,VMAGLP1)
      VYCOR = -OMEGM*R1/3600.
      DELVLP1 = QSQRT(VXLP1**2.+(VYLP1 - VYCOR)**2.+VZLP1**2.)
```

```
C     GAMA IS NEGATIVE AT SOI, POSITIVE AT L1 FOR OUTBOUND TRA-
C     JECTORIES, REVERSED FOR INBOUND FLIGHT.  NEED TO CHANGE
C     SIGN FOR SOI CALCULATIONS
      GAMAM = -GAMAM
      RETURN
      END

       SUBROUTINE XYZPOS(RRX,ALAT,ALON,XX,YX,ZX)
       IMPLICIT REAL * 16 (A-Z)
       XX = -RRX*COS(ALAT)*COS(ALON)+RREM
       YX = -RRX*COS(ALAT)*SIN(ALON)
       ZX =  RRX*SIN(ALAT)
       RETURN
       END


       SUBROUTINE POSVELMO(TIM,RRM,XDLO,YDLO)
       IMPLICIT REAL * 16 (A-H,O-Z)
       DELT = 0.5/36525./24./3600.
       T1 = TIM-DELT
       CALL MOON(T1,RAM,DECM,RM1)
       T2 = TIM+DELT
       CALL MOON(T2,RAM,DECM,RM2)
       XDLO = (RM2-RM1)*20925741.
       RRM = (RM2+RM1)/2.
       RRM = RRM
       RRMB = RRM -7.412789E-01
       YDLO = 200570.2/RRMB
       RETURN
       END



       SUBROUTINE MOON (T, RAM, DECM, RM)
C      FINDS LOCATION OF MOON IN EQUATORIAL COORDS. AT ANY TIM
C      REF:     '87 ASTRONOMICAL ALMANAC
C               T IS JULIAN CENTURIES SINCE YEAR 2000
C               LAM IS MOON'S ECLIPTIC LONGITUDE
C               BETA IS MOON'S ECLIPTIC LATITUDE
C               PIE IS HORIZONTAL PARALLAX
C               RM IS DIST. TO MOON IN EARTH RADII
C               RAM IS RT. ASCENSION OF MOON
C               DECM IS MOON'S DECLINATION
C               SD IS SEMIDIAMETER OF MOON'S ORBIT

      IMPLICIT REAL * 16 (A-Z)
C      PRINT *, ' MOON'
```

```
      P = 3.1415926535
      C = P / 180

      LAM = C*218.32+C*481267.883*T+C* 6.29 * QSIN(C * 134.9 + C *
     *477198.85 * T) - C * 1.27 * QSIN(C * 259.2 - C * 413335.38 *
     *T) + c * .66 * QSIN(C * 235.7 + c * 890534.23*T)

      LAM = LAM + c * .21 * QSIN(c * 269.9 + c * 954397.7*T) - c *
     *.19 *  QSIN(c * 357.5 + c * 35999.05 * T) - c * .11 *
     *SIN(c * 186.6 + c * 966404.05*T)

      beta = c*5.13*QSIN(c*93.3 + c * 483202.03 * T) + c *
     * .28 * QSIN(C * 228.2 +  C * 960400.87 * T) -c*.28*QSIN
     *(c*318.3+c* 60003.18*T)-c*.17*QSIN(c*217.6-c*407332.2 * T)

      pie = c*.9508+c*.0518*COS(c*134.9 + c * 477198.85 * T) +
     *c*.0095*QCOS(c*259.2-c*413335.38*T)+c* .0078*COS(c * 23
     *5.7+c*890534.23*T)+c*.0028*QCOS(c*269.9+c*954397.7* T)

      SD = .2725 * pie
      RM = 1. / QSIN(pie)

      l      = QCOS(beta)  * QCOS(LAM)
      M      = .9175 * QCOS(beta) * QSIN(LAM)  - .3978 * QSIN(beta)
      n      = .3978 * QCOS(beta)  * QSIN(LAM)  + .9175 * QSIN(beta)
      RAM    = QATAN2(M, l)
      DECM   = QASIN(n)
      RETURN
      END

      SUBROUTINE GAMACALC(RPX,VV,RRX,CMUX,COSGAMX,VPX,VCIRX,
     *TIMX,TRAJ,DPR,ALAT,ALON,FTNM)
      IMPLICIT REAL * 16 (A-H, O-Z)
      CHARACTER*10 TRAJ
      IHYPER = 1
      TRAJ = 'ELIPT     '
      QQX = RRX*VV**2/CMUX
      IF(QQX-2 .LT. 1.0E-06) QQX = QQX - 1.0E-06
      IF (QQX .GT. 2.) THEN
        IHYPER = -1
        TRAJ = 'HYPER     '
C       PRINT *,' ***** TRAJECTORY IS HYPERBOLIC '
      ENDIF
      AAX = RRX/(2.-QQX)
      IF (AAX. GT . 1.0E12 .OR. AAX. LT. -1.0E12) AAX = -1.0E12
      EEX        = 1.-RPX/AAX
      PPX        = AAX*(1. -EEX ** 2.)
      COSGAMX    = QSQRT(RPX/RRX*(1+EEX)/QQX)
      IF (COSGAMX .GT. 1.) THEN
```

```
        TIMX = 0.
        RETURN
      ENDIF
      GAMAX      = QACOS(COSGAMX)
      VPX        = QSQRT(CMUX*(1+EEX)/RPX)
      VCIRX      = QSQRT(CMUX/RPX)
      COSTHETAX = (PPX/RRX-1)/EEX
      THETAX     = QACOS(COSTHETAX)
      COSAEX     = (EEX+COSTHETAX)/(1+EEX*COSTHETAX)
      ERRRP      = RRX-AAX*(1+EEX)
      IF(ERRRP .GT. 0. .AND. AAX .GT. 0.) THEN
        WRITE (IS,537) ERRRP/6076.1155, ALAT*DPR, ALON*DPR
 537    FORMAT (' RADIUS > APOGEE BY NMI ',F7.5, F8.0, F7.5)
        WRITE (IS,547) QQX, AAX/FNTM, EEX, GAMAX*DPR, VPX
 547    FORMAT (' QQX = ',F7.5,' AAX = ',F8.0,' EEX = ',F7.5,
     *  ' GAMAX = ', F5.1,' VPX = ',F7.1)
      ENDIF
      IF(QQX .GT. 2.) GOTO 101
C       CALC FLIGHT TIM FOR ELIPTICAL ORBITS
        IF (ERRRP .GT. 0.) THEN
          TIMX = 0.
          GOTO 199
        ENDIF
        AEX    = QACOS(COSAEX)
        SINAEX = QSQRT(1-COSAEX**2.)
        TIMX   = QSQRT(AAX ** 3. / CMUX)*(AEX-EEX*SINAEX)/3600.
        GOTO 199
 101  CONTINUE
C     CALC FLIGHT TIM FOR HYPERBOLIC ORBITS
      COSHF = COSAEX
      SINHF = QSQRT(COSHF**2.-1.)
      FFX   = QLOG(COSHF+QSQRT(COSHF**2.-1.))
      TIMX = QSQRT(-1. *AAX*AAX*AAX/CMUX)*(EEX*SINHF-FFX)/3600.
 199  CONTINUE
      RETURN
      END


      SUBROUTINE VELTRANS(VEL,GAMA,AZM,ALAT,ALON,VXX,VYX,VZX,VMAG,
     *                      DPR)
      IMPLICIT REAL * 16 (A-Z)
      RRD   = VEL * QSIN (GAMA )
      RPHID = VEL * QCOS (GAMA )
      VLON  = RPHID * QSIN (AZM )
      VLAT  = RPHID * QCOS (AZM )
      VXR   = -RRD * QCOS (ALAT ) * QCOS (ALON )
      VYR   = -RRD * QCOS (ALAT ) * QSIN (ALON )
      VZR   =  RRD * QSIN (ALAT )
```

```
VXLA  = VLAT * QSIN (ALAT ) * QCOS (ALON )
VYLA  = VLAT * QSIN (ALAT ) * QSIN (ALON )
VZLA  = VLAT * QCOS (ALAT )
VXLO  = VLON * QSIN (ALON )
VYLO = -VLON * QCOS (ALON )
VZLO = 0.0
VXX  = VXR + VXLA + VXLO
VYX  = VYR + VYLA + VYLO
VZX  = VZR + VZLA + VZLO
VMAG = QSQRT (VXX **2. + VYX ** 2.+ VZX ** 2. )
RETURN
END
```

Appendix C. Program Variables

INPUT VARIABLE	DESCRIPTION
AINCEO	Earth orbit inclination (degrees). This is the angle between the plane of the low Earth orbit and the plane of the Moon's orbit about the Earth.
ALATI	Initial Sphere of Influence longitude for map (degrees)
ALONI	Initial Sphere of Influence latitude for map (degrees)
DELLAT	Incremental latitude for map (degrees)
DELLON	Incremental longitude for map (degrees)
FTIM	Flight time for trajectory (hours)
HPE	Holding perigee altitude of Earth orbit (nautical miles)
HPM	Perigee altitude of Lunar "flyby" trajectory (nautical miles)
MD	Leg of trip for which to perform calculations (OUTBOUND or RETURN)
TIMJ	Earth departure Julian date (where January 1, 2000 is day 2,451,545. Refer to Section C of "The Astronomical Almanac of the Year 1988").

CONSTANT	VALUE	DESCRIPTION
C	$\Pi/180$	Degrees per radian (deg./rad.)
CMUE	1.407647E+16	Gravitational parameter of the Earth (ft^3/sec^2)
CMUM	1.731432E+14	Gravitational parameter of the Moon (ft^3/sec^2)
DPR	57.29578	Degrees per radian (deg./rad.)
FTM	0.3048	Meters per foot (m/ft)
FTNM	6,076.115	Feet per nautical mile (ft/nmi)
P	3.1415927	Π (dimensionless)
PI	3.141593	Π (dimensionless)
REE	20,925,741	Radius of the Earth (ft)
REMO	5,703,900	Radius of the Moon (ft)
RREM	1,261,152,353	Earth-Moon distance of centers (ft)

VARIABLE	DESCRIPTION
AAX	Semi-major axis of one of the transfer orbits
AEX	Eccentric anomaly of one of the transfer orbits
AIM	Inclination of "flyby" trajectory
AINC	L1 orbit inclination in radians (always 0)
AINCE	Earth orbit inclination in radians
AINCEP	Earth orbit inclination in radians
AINCM	"Flyby" orbit inclination (degrees)
AINCMP	"Flyby" orbit inclination (degrees) for the SOI point associated with minimum ΔV.
ALAT	Initial latitude for map (radians)
ALATE	Latitude of SOI point, measured from the Earth
ALATL1	Latitude of L1, measured from the Moon (0°) (Moon-fixed)
ALATLP1	Latitude of L1, measured from the Moon (0°)
ALATMIN	Latitude of SOI point associated with the minimum total ΔV
ALATP()	ALAT in degrees
ALATSIM	Latitude of SOI point associated with the minimum SOI ΔV
ALATX	Latitude of the SOI point, measured from the Earth
ALON	Initial longitude for map (radians)
ALONE	Longitude of SOI point, measured from zero longitude at Earth (-X direction)
ALONL1	Longitude of L1, measured from the Moon (Moon-fixed)
ALONLP1	Longitude of L1, measured from the Moon (180°)
ALONMIN	Longitude of SOI point associated with the minimum total ΔV
ALONO()	Variable that contains the print matrix column headings (longitudes)
ALONSIM	Longitude of SOI point associated with the minimum SOI ΔV

ALONX	Longitude of the SOI point, measured from the Earth-Moon line at the Earth
AMAG	Magnitude of position vector cross-product (R2 X R1) used to calculate "flyby" inclination
ANGA	The angle between the Earth-to-Moon line and the Earth-to-SOI-point line
ANODE	The longitude of the ascending or decending node of "flyby" trajectory
ANODEE	Longitude of the Earth ascending or descending node
ANODEEP	Longitude of the Earth ascending or descending node associated with minimum ΔV
ANODEM	Longitude of the Lunar ascending or descending node
ANODEMP	Longitude of the Lunar ascending or descending node associated with the minimum ΔV
APHIL1	Angle between line of nodes and L1
APHIM	Angle between line of nodes and Moon-to-SOI line
AVM	Angle between the Lunar node and the projection of the SOI point onto the Earth-Moon plane (υ)
AZM	Azimuth of the SOI point
AZME	Azimuth (from the Earth) of the SOI point for Earth-SOI trajectory
AZMEP	Azimuth (from the Earth) of the SOI point for Earth-SOI trajectory associated with minimum ΔV
AZMLP1	Azimuth of "flyby" orbit at L1
AZMM	Azimuth (from the Moon) of the SOI point for "flyby" trajectory
AZMMP	Azimuth (from the Moon) of the SOI point for "flyby" trajectory with minimum ΔV
BETA	Moon's ecliptic latitude
CMUX	Earth or Moon gravitational parameter
COSAEX	Cosine of the eccentric anomaly of one of the transfer orbits
COSAINC	Cosine of the L1 orbit inclination (always 1)
COSALAT	Cosine of the SOI point latitude

COSALON	Cosine of the SOI point longitude
COSANGA	Cosine of the angle between the Earth-to-Moon line and the Earth-to-SOI-point line
COSAZM	Cosine of the azimuth angle at the SOI point or L1 for the "flyby" trajectory
COSGAMA	Cosine of flight path angle at SOI for "flyby" trajectory
COSGAME	Cosine of the flight path angle at SOI of the Earth-to-SOI trajectory
COSGAMX	Cosine of the flight path angle at SOI of one of the transfer orbits
COSHF	Hyperbolic cosine of the eccentric anomaly of one of the transfer orbits
COSPHI	Cosine of the angle between the Moon-to-Earth line and the Moon-to-SOI-point line
COSPHIL1	Cosine of angle between line of nodes and L1
COSPHIM	Cosine of the angle between the line of nodes and the Moon-to-SOI line
COSTHETAM	Cosine of the true anomaly of the "flyby" orbit
COSTHETAX	Cosine of the true anomaly of one of the transfer orbits at SOI
COS2THETA	Cosine of two times the true amomaly of the "flyby"
DATP	Today's date
DECM	Declination of the Moon
DELT	A fraction of TIM that represents a half-second
DELV(,)	Total ΔV for matrix of SOI longitudes and latitudes
DELVEL	ΔV at the SOI point to patch the "flyby" trajectory with the Earth-to-SOI trajectory
DELVELP	ΔV at the SOI point to patch the "flyby" trajectory with the Earth-to-SOI trajectory associated with minimum ΔV
DELVELE	Estimated ΔV required for the Earth-to-SOI trajectory in order to satisfy the time-of-flight requirement
DELVLP1	ΔV for circularization at L1
DELVLP1P	ΔV for circularization at L1 associated with minimum ΔV

DOT	Dot product of position vectors for L1 and SOI point (R1 R2)
DVCIRE	ΔV between Earth circular orbit and Earth-to-SOI trajectory
DVCIREP	ΔV between Earth circular orbit and Earth-to-SOI trajectory associated with minimum ΔV
DVE	Unused plane change variable
DVMIN	Hold variable for minimum total ΔV
DVSIM	Hold variable for minimum SOI ΔV
DVTOTAL	Total ΔV for flight
DVTOTALP	Total ΔV for flight associated with minimum ΔV
DVTSAV	Temporary storage for total ΔV
EEM	Eccentricity of the "flyby" orbit
EEX	Eccentricity of one of the transfer orbits
ERRRP	Difference between orbital range at SOI and apogee range
FFX	Hyperbolic eccentric anomaly
GAMA	Flight path at SOI of one of the transfer orbits
GAMAE	Flight path at SOI of Earth-to SOI trajectory
GAMAEP	Flight path at SOI of Earth-to SOI trajectory associated with minimum ΔV
GAMAM	Flight path at SOI of "flyby" trajectory
GAMAMP	Flight path at SOI of "flyby" trajectory with minimum ΔV
GAMAX	Flight path angle at SOI of one of the transfer orbits
HEAD	Heading that indicates leg of journey (e.g., "Earth to L1 Flyby" for outbound)
ICALL	Flag to track occurance of call to in-program subroutine.
II	Counter (1-19), representing increments in latitude for the output matrices
IHYPER	Indicator that describes whether an orbit is hyperbolic
IP	Output number indicating output to file

IPRINT	Flag for print block that contains matrix column headers
IS	Output number indicating output to screen
L	A geocentric direction cosine
LAM	Moon's ecliptic longitude
M	A geocentric direction cosine
MOOD	Leg of journey (+1: outbound)
N	A geocentric direction cosine
NN	Counter (1-10) representing increments in longitude for the output matrices
NPI	Counter for the 19 matrix rows during report printing
OMEGE	Argument of perigee for Earth-SOI trajectory
OMEGM	Angular velocity of Moon
PAGE1()	Total ΔV for a given SOI longitude and latitude. Cell values for Report #1 matrix
PAGE2(,)	SOI ΔV for a given SOI longitude and latitude. Cell values for Report #2 matrix
PAGE3(,)	Inclination of "flyby" trajectory for a given SOI longitude and latitude. Cell values for Report #3 matrix
PHIE	Used to determine the azimuth of the SOI point
PIE	Horizontal parallax
PPM	Semi-latus rectum of the "flyby" orbit
PPX	Semi-latus rectum of one of the transfer orbits
QQEMIN	Vis-viva parameter for the Earth-to-SOI-point trajectory
QQM	Vis-viva parameter for the "flyby" orbit
QQX	Vis-viva parameter for an orbit
R1	Distance of L1 and SOI surface from center of Moon (15% of Earth-Moon distance)
RAM	Right ascension of the Moon
RM	Distance from the Earth to the Moon in Earth radii

RMR	Ratio of Earth-Moon distance to Moon-SOI distance
RM1	Distance from the Earth to the Moon in Earth radii
RM2	Distance from the Earth to the Moon in Earth radii
RPE	Distance from Earth's center to Earth perigee orbit
RPHID	Orbital path component of the velocity vector
RPM	Distance from Moon's center to Lunar "flyby" perigee
RPX	Distance from Earth center to perigee
RRD	Radial component of the velocity vector
RRE	Distance from the Earth to the SOI point
RREMER	Moon's distance from the Earth (Earth radii)
RRM	Distance from the Moon to the SOI point, when the Moon is at its mean distance from the Earth
RRMB	Distance of the Moon from the Earth-Moon baricenter
RRX	Distance from Earth to SOI
SD	Semi-diameter of the Moon's orbit
SINAEX	Sine of the eccentric anomaly of one of the transfer orbits
SINAINC	Sine of the Lunar orbit inclination
SINALAT	Sine of the latitude of the SOI point
SINALON	Sine of the longitude of the SOI point
SINANGA	Sine of the angle between the Earth-to-Moon line and the Earth-to-SOI point line
SINAZM	Sine of the azimuth angle at the SOI point or L1 for the "flyby" trajectory
SINHF	Hyperbolic sine of the eccentric anomaly of one of the transfer orbits
T	Number of Julian centuries since the year 2000 AD
THETA	True anomaly of "flyby" at the SOI
THETAX	True anomaly of one of the transfer orbits at SOI
TIEM	Earth-to-SOI time of flight (seconds)

TIEMP	Earth-to-SOI time of flight (seconds) associated with minimum ΔV
TIEMS	Temporary storage for Earth-to-SOI time of flight
TIM	Time of arrival at SOI point from Earth, in centuries since the year 2000 AD
TIME1	Earth-to-SOI time of flight (seconds)
TIME2	Earth-to-SOI time of flight (seconds)
TIEMM	Time of flight from SOI to L1
TIEMMP	Time of flight from SOI to L1 associated with minimum ΔV
TIEMMS	Temporary storage for SOI to L1 time of flight
TIMP	Time Now
TIMT	Total time of flight
TIMX	Time of perigee passage for "flyby"
TRAJ	Text that describes whether an orbit is hyperbolic or elliptical
TRAJE	Text that describes whether the Earth-to-SOI trajectory is hyperbolic or elliptical
TRAJM	Text that describes whether the "flyby" trajectory is hyperbolic or elliptical
T1	One-half second before TIM
T2	One-half second after TIM
VCIRE	Velocity of Earth circular orbit
VCIRX	Velocity of the Earth circular orbit
VEL	Velocity at SOI of one of the transfer orbits
VELE	Velocity at SOI of the Earth-to-SOI trajectory
VELEP	Velocity at SOI of the Earth-to-SOI trajectory associated with minimum ΔV
VELE2	Ten feet per second more than VELE
VELEMIN	Minimum velocity required such that apogee of the trajectory is just at SOI
VELM	Velocity at SOI of the "flyby" trajectory
VELMMIN	Velocity at SOI of the "flyby" trajectory associated with the minimum total ΔV

VELMOUT(,)	Velocity at SOI of the "flyby" trajectory, for a given SOI longitude and latitude
VELMSIM	Velocity at SOI of the "flyby" trajectory associated with the minimum SOI ΔV
VLAT	Latitude component of the velocity vector
VLON	Longitude component of the velocity vector
VMAG	Velocity vector magnitude
VMAGLP1	Velocity vector magnitude at L1 for "flyby" trajectory
VMAGM	Velocity vector magnitude at SOI for "flyby" trajectory
VMAGMP	Velocity vector magnitude at SOI for "flyby" trajectory with minimum ΔV
VPE	Perigee velocity of the Earth-to-SOI trajectory
VPM	Perigee velocity of the "flyby" trajectory
VPX	Perigee velocity for one of the transfer orbits
VV	Velocity at SOI of one of the transfer orbits
VVMINE	Velocity at SOI point (apogee) of Earth-to-SOI trajectory
VXE	X-coordinate of velocity vector at SOI for Earth-to-SOI trajectory
VXEP	X-coordinate of velocity vector at SOI for Earth-to-SOI trajectory associated with minimum ΔV
VXLA	X-component of the latitude component of the velocity vector
VXLO	X-component of the longitude component of the velocity vector
VXLP1	X-component of the velocity vector at L1 for "flyby"
VXM	X-coordinate of velocity vector at SOI for "flyby" trajectory
VXMP	X-coordinate of velocity vector at SOI for "flyby" trajectory with minimum ΔV
VXR	X-component of the radial component of the velocity vector
VXX	Total X-component of the velocity vector
VYCOR	Y-component of velocity for orbit at L1 (no X- or Z- component)
VYE	Y-coordinate of velocity vector at SOI for Earth-to-SOI trajectory

VYEP	Y-coordinate of velocity vector at SOI for Earth-to-SOI trajectory associated with minimum ΔV
VYLA	Y-component of the latitude component of the velocity vector
VYLO	Y-component of the longitude component of the velocity vector
VYLP1	Y-component of the velocity vector at L1 for "flyby"
VYM	Y-coordinate of velocity vector at SOI for "flyby" trajectory
VYMP	Y-coordinate of velocity vector at SOI for "flyby" trajectory with minimum ΔV
VYR	Y-component of the radial component of the velocity vector
VYX	Total Y-component of the velocity vector
VZE	Z-coordinate of velocity vector at SOI for Earth-to-SOI trajectory
VZEP	Z-coordinate of velocity vector at SOI for Earth-to-SOI trajectory associated with minimum ΔV
VZLA	Z-component of the latitude component of the velocity vector
VZLO	Z-component of the longitude component of the velocity vector
VZLP1	Z-component of the velocity vector at L1 for "flyby"
VZM	Z-coordinate of velocity vector at SOI for "flyby" trajectory
VZMP	Z-coordinate of velocity vector at SOI for "flyby" trajectory with minimum ΔV
VZR	Z-component of the radial component of the velocity vector
VZX	Total Z-component of the velocity vector
X1	X-component of the distance form the Moon to L1
X2	X-component of the distance from the Moon to the SOI penetration point
X3	X-component of cross-product (R1 X R2)
XDLO	X-coordinate of the velocity of the Moon
XX	Distance in the X-direction from the Earth to the SOI point's X-coordinate
XXM	Distance in the X-direction from the Moon to the SOI point's X-coordinate
Y1	Y-component of the distance form the Moon to L1

Y2	Y-component of the distance from the Moon to the SOI penetration point
Y3	Y-component of cross-product (R1 X R2)
YDLO	Y-coordinate of the velocity of the Moon
YY	Distance in the Y-direction from the Earth-Moon line to the SOI point's Y-coordinate
YYM	Distance in the Y-direction from the Moon to the SOI point's Y-coordinate
Z1	Z-component of the distance form the Moon to L1
Z2	Z-component of the distance from the Moon to the SOI penetration point
Z3	Z-component of cross-product (R1 X R2)
ZZ	Distance in the Z-direction from the Earth-Moon plane to the SOI point

Appendix D. Detailed Program Description

This section is a line-by-line description of the lines of code in the main program of LP1. Refer to Appendix B for the actual code listing, and figures K-1 through K-3 for supporting illustrations.

1. Declare the matrices DELV, VELMOUT, PAGE1, PAGE2, PAGE3, PAGE4, PAGE5, PAGE6, ALONO, and ALATP.

2. Open the output file (LP1.OUT).

3. Define the program constants.

4. Read the program inputs.

    A. MD (leg of trip for which to perform calculations; outbound or return)

    B. HPE (perigee altitude of Earth circular orbit)

    C. HPM (perigee altitude of Lunar "flyby" trajectory)

    D. TIMJ (Earth departure Julian date. Default is 2451545, representing Jan. 1, 2000 AD)

    E. ALONI, DELLON (Initial longitude and increment for output map)

    F. ALATI, DELLAT (Initial latitude and increment for output map)

    G. AINCEO (Angle between the plane of the low Earth orbit and the Earth-Moon plane)

    H. FTIM (flight time)

5. Calculate the distance from the Earth's center to Earth perigee orbit.
   RPE = HPE * FTNM + REE

6. Calculate the distance from the Moon's center to Lunar perigee orbit.
   RPM = HPM * FTNM + REMO

7. Store the inclinations in temporary variables (AINCE and AINC).

8. Record the current date and time (DATP and TIMP).

9. Print the headings for Report #1, "Velocity Map for Inbound/Outbound Trajectories".

10. Echo back the input values onto Report #1.

11. Initialize the velocity map matrix coordinates to 1,1 (II = rows, or latitude increments; NN = columns, or longitude increments). Set the flag IPRINT to zero, which has the

effect of causing the longitude increments to be printed as a subheading. Initialize the minimum ΔV hold variable (DVMIN) to 99999.

12. Calculate the orbital velocity of the Moon.

$$YDLO = \mu E / RREM$$

13. Convert the initial map latitude and longitude from degrees to radians (ALAT and ALON).

14. Initialize DVSIM at 99999. This variable will be used to hold the lowest ΔV for the trajectory, selected from all combinations of latitude and longitude.

15. Initialize the current Total ΔV matrix cell (DELV) to 99999.

16. Initialize with zeros the print line variables for the current cell on each report (PAGE1, PAGE2, PAGE3, PAGE4, PAGE5, PAGE6).

17. Call the flyby subroutine to calculate the characteristics for a "flyby" trajectory between the current SOI point and L1. Pass input values for SOI latitude and longitude, Earth-Moon distance, and perigee altitude; and return output values for flight path angle and velocity (same for SOI point and L1), perigee velocity, time of flight, delta velocity needed to circularize at L1 (outbound) or initiate the "flyby" (return), azimuth angle, inclination of the "flyby" trajectory, and the angle between the lunar nodes and the SOI point projection.

18. Call the in-program subroutine given the current SOI conditions for the "flyby" trajectory (latitude, longitude, inclination, flight path, azimuth, and velocity) and iterate for a transfer orbit between LEO and the SOI point (with a velocity vector correction at the SOI) and determine the total ΔV and total flight time needed.

19. Store the total ΔV, SOI ΔV, and "flyby" inclination in arrays for output.

20. If all the matrix columns have been processed for this latitude, and if this has been the first row of the matrix, print the matrix column headings for Report #1.

21. If all ten matrix columns have not been processed for this latitude, increment the column counter by one and increment the longitude by the amount specified by the user in the inputs. Return to step 15 to process the next longitude/latitude combination.

22. If all ten matrix columns have been processed for this latitude, print the matrix row for this latitude for Report #1.

23. If all 19 matrix rows have not been processed, increment the row counter by one, increment the latitude by the amount specified by the user in the inputs, reset the column counter to one, reset the longitude to the initial input longitude, and return to step 15 to process the next longitude/latitude combination.

24. If all 19 matrix rows have been processed, continue with the following steps.

25. Print the bottom section of Report #1, and the remaining five reports.

Appendix E. In-Program Subroutine Description

This section is a line-by-line description of the lines of code in the subroutine that is imbedded in program LP1, beginning at line #25. Refer to Appendix B for the actual code listing, and figures K-2 through K-4 for supporting illustrations.

I. Establish the key distances and angles of the current SOI point from the Earth and Moon.

A. Calculate the distance from the Moon to the SOI point.

1. Define variables for the sine and cosine of the SOI latitude and longitude (COSALAT, COSALON, SINALAT, SINALON).

2. Given a point on the SOI defined by the latitude and longitude, define the angle between the Moon-to-Earth line and the Moon-to-SOI line.
$\cos(\ ) = \cos(lat) * \cos(lon)$

3. Define variables for the sine and cosine of the Lunar orbit inclination (COSAINC and SINAINC).

4. Calculate the ratio of Earth-Moon distance to Moon-SOI distance (RMR = REM / RM).

a. $RRE^2 = RRM^2 + RREM^2 - 2 * RRM * RREM * \cos(\ )$

b. $\mu E/RRE^2 = \mu M/RRM^2 \quad \therefore \quad RRE^2/RRM^2 = \mu E/\mu M$

(because μ/R^2 = gravitational acceleration, and by definition, at the sphere of influence, acceleration towards the Earth equals acceleration towards the Moon).

c. $RRE^2/RRM^2 = \mu E/\mu M = 1 + RREM^2/RRM^2 - 2*(RREM/RRM)*\cos(\ )$
$= 1 + RMR^2 - 2*RMR*\cos(\ )$

d. $0 = -(\mu E/\mu M) + 1 + RMR^2 - 2*RMR*\cos(\ )$
$= RMR^2 - 2*RMR*\cos(\ ) + 1 - (\mu E/\mu M)$

e. For the quadratic formula:
$a = 1, \quad b = -2*\cos(\ ), \quad c = 1 - (\mu E/\mu M)$

$RMR = (2*\cos(\ ) \pm \ 4\cos^2 \ - 4(1 - \mu E/\mu M)\ ) / 2$

$RMR = \cos(\ ) + \ \cos^2 \ - 1 + \mu E/\mu M$

5. Determine the distance from the Moon to the SOI.
RRM = RREM / RMR

B. Determine an Earth-based rectangular coordinate system such that the X-direction is on the line from the Earth to the Moon; the Y-direction is in the direction of the Moon's orbit; and the Z-direction is perpendicular to X and Y in right-hand coordinates. Project the SOI point onto the X-Y plane (Earth-Moon plane) in order to calculate the X- and Y-coordinates.

1. Determine the distance from the Moon to the X-intercept.
   $$XXM = RRM * \cos(\ ) = RRM * COSALAT * COSALON$$

2. Determine the distance from the Earth to the X-intercept.
   $$XX = RREM - XXM$$

3. Determine the Y-coordinate. Note that longitude increases in the negative Y direction.
   $$YY = -RRM * COSALAT * SINALON$$

4. Determine the Z-coordinate.
   $$ZZ = RRM * SINALAT$$

5. Determine the distance from the Moon to the Y-intercept.
   $$YYM = -YY$$

C. Calculate the distance from the Earth to the SOI point.
$$RRE = \ XX^2 + YY^2 + ZZ^2$$

D. Identify the angle between the Earth-to-Moon line and the Earth-to-SOI line.

1. $\cos(ANGA) = XX / RRE$

2. $\sin(ANGA) = \ YY^2 + ZZ^2 \ / RRE$

3. $ANGA = \tan^{-1}(\sin(ANGA) / \cos(ANGA))$

E. Determine the Earth-based latitude and longitude of the SOI point.

1. $ALONX = \tan^{-1}(YY / XX)$

2. $ALATX = \tan^{-1} (ZZ / \ XX^2 + YY^2)$

F. Determine the Earth ascending node for the orbit in which the trans-SOI burn will occur.

1. Given the latitude of the burn at Earth (negative SOI latitude) and the inclination of the Earth orbit, spherical coordinate trigonometry states that the longitude from the node to the burn point is
   $$\upsilon = \sin^{-1} (\tan(-ALATX) / \tan(AINCE))$$

2. Longitude of the burn point (measured from the Earth-Moon line) is ALONX.

3. The node is calculated by subtracting the longitude in (a) above from the longitude in (b).

$$ANODEE = ALONX - \sin^{-1}(\tan(-ALATX) / \tan(AINCE))$$

II. For the SOI-to-L1 trajectory, calculate Lunar node positions, and velocity vector at the current SOI point with respect to Earth.

A. Determine the node.

$$ANODEM = ALON - AVM$$

B. Determine the components of the velocity vector at the SOI point.

1. Call the subroutine VELTRANS to determine the X-, Y-, and Z-components of the velocity vector, from the viewpoint of the moon.

2. Add the X- and Y-components of the Moon's velocity to determine the SOI velocity from the viewpoint of the Earth.

$$VXM = VXM + XDLO \qquad VYM = VYM + YDLO$$

III. For the Earth-to-SOI trajectory, calculate velocity at the SOI point, ΔV at Earth circular orbit, and time of flight.

A. Modify the velocity at SOI, if necessary, to bring the trajectory up to at least the minimum velocity required to intercept the SOI.

1. Calculate the vis-viva parameter QQEMIN for the Earth-to-SOI trajectory.

$$QQEMIN = 2 - (RRE / a) \quad \text{where } a = (RPE + RRE) / 2$$
$$QQEMIN = 2 - ((2 * RRE) / (RPE + RRE))$$
$$QQEMIN = 2 * (1 - (RRE / (RPE + RRE)))$$
$$QQEMIN = 2 * (RPE / (RPE + RRE))$$

2. Calculate the minimum velocity required, such that apogee is just at SOI.

$$QQEMIN \equiv (V^2 * RRE) / CMUE$$
$$\therefore VELEMIN = \quad (QQEMIN * CMUE) / RRE$$

3. Increase the velocity slightly so that the calculations converge.

4. Compare the most recently calculated SOI velocity (VELE) with the minimum velocity. If it is too low, bring it up to the minimum.

B. Temporarily store (until step IV-D) the calculated Earth-to-SOI time of flight.

C. Further increase the velocity at SOI, if necessary, to meet the Earth-to-SOI time of flight requirement (but capping the ΔV at 500 ft/sec). Calculate the new SOI flight path angle, ΔV at Earth circular orbit, and time of flight.

1. Call the subroutine GAMACALC to determine the time of flight from Earth perigee to SOI (TIMEE1), given the velocity VELE.

2. Increase the velocity by 10 ft/sec, and call GAMACALC again to determine the new time of flight (TIMEE2).

3. For the Earth-to-SOI leg, determine the ratio of time-of-flight shortfall to the time-of-flight increase that was due to the 10 ft/sec increase in velocity.

   Shortfall/Increase = (FTIME-TIMEE1-TIMEM) / (TIMEE2 - TIMEE1)

   Apply this ratio to the increased velocity of 10 ft/sec to estimate the ΔV necessary to meet the required time of flight.

   DELVELE = 10 * (FTIME - TIMEE1 - TIMEM) / (TIMEE2 - TIMEE1)

4. Cap this ΔV at 500 ft/sec, and add it to the original velocity.

5. Call the subroutine GAMACALC to determine the new Earth-to-SOI time of flight for the revised velocity.

D. Compare the total Earth-to-Moon time of flight to the required time of flight. If the total is short, return to step III-C.

E. Determine the components of the velocity vector at the SOI point.

1. Calculate the flight path angle at SOI (GAMAE) for the Earth-to-SOI trajectory. The cosine of the flight path angle will be between zero and one, corresponding to angles between zero and 90. In reality, Moon-to-SOI flight path angles will range from zero to 90, but SOI-to-Moon angles will range from zero to -90. Gamma must be derived from cos(GAM) and multiplied by -MOOD (from the input: +1 for outbound and -1 for return) to get the correct flight path angle. The arctan function is used instead of arccos to evaluate gamma because of that function's better debugging diagnostics capabilities.

   $GAMAE = -MOOD * \tan^{-1}(1 - COSGAME^2 / COSGAME)$

2. Determine the longitude of the SOI from the Earth's viewpoint.

   ALONE = ALONX + 180 (since zero longitude is on the Earth-Moon line on the side of the Earth <u>away</u> from the Moon).

3. Determine the latitude of the SOI point.

   $ALATE = \tan^{-1} (ZZ / \quad XX^2 + YY^2)$

4. Calculate the azimuth of the SOI point

   $AZME = \tan^{-1} (1/\tan(AINCE), \cos(PHIE))$

   where $PHIE = \Pi - \sin^{-1}((ZZ/RRE)/\sin(AINCE))$

5. Call the subroutine VELTRANS to determine the X-, Y-, and Z-components of velocity at SOI for the Earth-to-SOI trajectory.

IV. Combine the SOI-to-Moon calculations with the Earth-to-SOI calculations to identify total flight characteristics.

A. Calculate total ΔV

1. Calculate ΔV at the SOI point required to patch the Earth-to-SOI trajectory into the SOI-to-L1 trajectory. (This value is calculated by determining the difference between each of the components of the two trajectories).

$$DELVEL = (VXE - VXM)^2 + (VYE - VYM)^2 + (VZE - VZM)^2$$

2. Calculate ΔV at Earth perigee.

DVCIRE = VPE - VCIRE

3. Save the old total ΔV.

DVTSAV = DVTOTAL

4. Calculate the new total ΔV.

DVTOTAL = DELVEL + DELVLP1 + DVCIRE

B. Determine the distance from the Moon to the Earth for this trajectory just calculated. Use this value in subsequent iterations of the in-program subroutine, if necessary.

1. Determine the time of arrival at SOI from Earth, in centuries since the year 2000 AD.

TIM = Departure date + Time of flight - 2000

where Departure date = TIMJ / 36525 days per century
Time of flight = TIEM hrs / 876600 hrs per century
The year 2000 = Day 2451545 / 36525 days per century.

2. Call the subroutine POSVELMO to determine the Moon's distance from the Earth and velocity at the time of arrival at SOI from the Earth.

3. Convert Lunar distance from Earth radii to feet.

RREM = RREMER * 20295741 ft/radii.

C. Compare the value of Earth-to-SOI time of flight saved in step III-B above with the new value of time of flight calculated in step III-C above. If the difference has not yet converged (to less than 0.1 hour), reiterate this in-program subroutine beginning at step I-A. Otherwise, continue.

D. If the newly calculated total ΔV is the smallest ΔV calculated so far, then store it in the variable DVMIN. Also store all the significant orbital characteristics of the trajectory for later use in the bottom section of Report #1.

E. If the newly calculated SOI ΔV is the smallest calculated so far, then store it in the variable DVSIM. Also store its corresponding SOI longitude (ALONSIM), SOI latitude (ALATSIM), and SOI velocity for the SOI-to-Moon trajectory (VELMSIM).

Appendix F. Subroutine FLYBY

The subroutine FLYBY receives the parameters SOI longitude (ALON) and latitude (ALAT), Earth-Moon distance, "flyby" perigee radius (RPM), and flight mode (MOOD). FLYBY contains the same list of constants as in the main program, plus the radius of the SOI (R1) and the mean angular speed of the Moon (OMEGM). It calculates the magnitude of the velocity at the SOI and at L1 (VELM), the flight path angle at the SOI and at L1 (GAMAM), the inclination of the "flyby" trajectory (AIM), the circularization ΔV needed at L1 (DELVLP1), the azimuth angle at the SOI (AZMM), the angle between the Lunar node and the SOI projection (AVM), and the flight time between SOI and L1 (TIEMM).

As mentioned briefly in the introduction, the FLYBY routine takes advantage of one simplification. By enlarging the SOI radius to include L1 from 11% of the Earth-Moon distance to 15%, the problem can be simplified considerably without significant error. Since the SOI penetration point is at the same distance from the Moon as L1, perigee passage will occur exactly half-way between the two points for any orbit containing them. Consequently, the true anomaly, flight path angle, and velocity will be the same at both the SOI and L1.

1. Initialize constants.

2. Determine location of L1 in Lunar relative coordinates. Remember, L1 rotates with the Moon, and therefore is affected by "flyby" time. X-axis is measured form Lunar center to Earth. Y-axis is measured in the opposite direction of the Moon's velocity.
   X1 = -R1 * cos(OMEGM * TIEMM)
   Y1 = -R1 * sin(OMEGM * TIEMM) * MOOD
   Z1 = 0

3. Determine location of SOI point.
   X2 = R1 * cos(ALAT) * cos(ALON)
   Y2 = R1 * cos(ALAT) * sin(ALON)
   Z2 = R1 * sin(ALAT)

4. Calculate true anomaly.
   A. Dot product of position vectors for SOI point and L1.
      DOT = X1 * X2 + Y1 * Y2 + Z1 * Z2

   B. From geometry:
      $\cos(2\Theta) = DOT/R1/R1$

   C. It follows:
      $\Theta = \arccos(\cos(2\Theta))/2$

5. Calculate eccentricity of "flyby" orbit using general conic equation.
   $EEM = (RPM/R1 - 1)/(\cos(\Theta) - RPM/R1)$

6. Calculate flight path angle.
   $\cos(\gamma) = (1 + EEM * \cos(\Theta))/ (1 + 2 * EEM * \cos(\Theta) + EEM^2)$
   $\gamma = \mathrm{arcos}(\cos(\gamma)) * MOOD$

7. Calculate semi-latus rectum.

$$PPM = RPM * (1 + EEM)$$

8. Calculate perigee velocity.

$$VPM = (\mu_m * (2/RPM + (EEM^2 - 1)/PPM))$$

9. Calculate velocity at SOI and L1.

$$VELM = (\mu_m * (2/R1 + (EEM^2 - 1)/PPM))$$

10. Calculate inclination by crossing position components (R2 X R1).

$$X3 = Y2 * Z1 - Z2 * Y1$$
$$Y3 = Z2 * X1 - X1 * X2$$
$$Z3 = X2 * Y1 - X1 * Y2$$
$$MAG = (X3 * X3 + Y3 * Y3 + Z3 * Z3)$$
$$IAM = \arccos(Z3/MAG)$$

11. Calculate the vis-viva parameter.

$$QQM = R1 * VELM * VELM/\mu_m$$

12. Calculate eccentric anomaly.

$$COSAEX = (EEM + \cos(\Theta))/(1 + EEM * \cos(\Theta))$$
$$AEX = \arccos(COSAEX)$$

13. Calculate semi-major axis.

$$AAX = R1/(2 - QQM)$$

14. Store old "flyby" time of flight.

$$TIEMMS = TIEMM$$

15. Calculate the time of perigee passage.

A. If the orbit is elliptical:

$$TIMX = (AAX\wedge 3/\mu_m) * (AEX - EEM * \sin(AEX)).$$

B. If the orbit is hyperbolic:

$$TIMX = (-AAX\wedge 3/\mu_m) * (EEM * \sinh(EEM) - \log(\cosh(EEM) + \sinh(EEM))).$$

16. Calculate time of flight for "flyby".

$$TIEMM = TIMX * 2$$

17. Calculate longitude of nodes.

$$ANODE = OMEGM * TIEMM$$

IF ((ALAT*MOOD) >= 0) then ANODE = ANODE + PI

18. Calculate longitude angle between nodes and the SOI projection.

$$AVM = ALON - ANODE$$

19. Calculate angle between nodes and SOI projection.
    COSPHIM = cos(ALAT) * (cos(ALODE) * cos(ALON) + sin(ANODE) * sin(ALON))
    APHIM = arccos(COSPHIM)

20. Calculate azimuth angle for R2 (at SOI).
    SINAZM = cos(AIM)/cos(ALAT)
    COSAZM = sin(AIM)*cos(APHIM)/cos(ALAT)
    AZMM = arctan2(SINAZM, COSAZM)

21. Calculate position for L1 (X-Y axes are fixed at Moon)
    ALATL1 = 0
    ALONL1 = arctan2(Y1, X1)

22. Calculate angle between nodes and L1.
    COSPHIL1 = cos(ALATL1) * (cos(ALODE) * cos(ALONL1) + sin(ANODE) * sin(ALOL1N))
    APHIL1 = arccos(COSPHIL1)

23. Calculate azimuth angle for R1 (at L1).
    SINAZM = cos(AIM)/cos(ALATL1)
    COSAZM = sin(AIM)*cos(APHIL1)/cos(ALATL1)
    AZMLP1 = arctan2(SINAZM, COSAZM)

24. If new "flyby" flight time is not very close to the saved value go back to step 2.

25. Set latitude/longitude position of L1 (X-Y axes are rotating)
    ALATLP1 = 0°
    ALONLP1 = 180°

26. Call Veltrans to establish the velocity components of L1 in Lunar-based rectangular components.

27. Set Y-component of velocity for L1 (X- and Z- components are 0).
    VYCOR = -OMEGM * R1/3600.

28. Calculate the ΔV needed to correct the velocity vector at L1 to that of L1's orbit (outbound) or the "flyby" to the SOI (return).
    $DELVLP1 = (VXLP^2 + (VYLP1 - VYCOR)^2 + VZLP1^2)$

29. Sign of flight path angle must be changed for SOI calculations.
    GAMAM = -GAMAM

Appendix G. Subroutine GAMACALC

The subroutine GAMACALC receives the parameters orbital perigee radius (RPX), orbital velocity at LP (VV), orbital radius at LP (RRX), and the gravitational parameter of the body being orbited (CMUX). It calculates and returns time of flight (TIMX), the cosine of the flight path angle (COSGAMX), velocity at periapses (VPX), circular velocity at periapses (VCIRX), true anomaly (THETAX), and an indicator describing whether the orbit is elliptical or hyperbolic (TRAJ$).

1. Initialize indicators to presume an elliptical orbit. (IHYPER, TRAJ$).

2. Calculate the vis-viva parameter
   QQX = (RRX * VELX^2) / CMUX.

3. If the orbit is just barely hyperbolic (QQX is within one-millionth of 2), force the calculation to consider it elliptical (reduce QQX to just under 2).

4. If the orbit is still hyperbolic, reset the indicators to show this. Print a message on the screen announcing a hyperbolic orbit.

5. Calculate the semi-major axis of the orbit
   AAX = RRX / (2-QQX).

6. If the semi-major axis is very large (greater than 10^12) or very small (less than -10^12), the orbit is trapped near a parabolic trajectory. Make it hyperbolic:
   AAX = -10^12.

7. Calculate the orbit eccentricity
   EEX = 1 - (RPX/AAX).

8. Calculate the semi-latus rectum
   PPX = AAX * (1 - EEX).

9. Calculate the flight path angle

   a. The angular momentum of the orbit at perigee is
      HP = RPX * VELPERIGEE * cos(GAMAX).
      But at perigee, GAMAX is zero, so
      HP = RPX * VELPERIGEE.

   b. At LP, angular momentum is
      HX = RRX * VELX * cos(GAMAX).

   c. Angular momentum is constant along a given orbit, so
      HP = HX
      RPX * VELPERIGEE = RRX * VELX * cos(GAMAX)
      GAMAX = arccos((RPX * VELPERIGEE) / (RRX * VELX))

d. The velocity at perigee is

VELPERIGEE = sqr((CMUX * RAPOGEE) / (AAX * RPX)).

Since RAPOGEE / AAX = (1 + EEX), then

VELPERIGEE = sqr (CMUX * (1 + EEX) / RPX) or

RPX * VELPERIGEE = sqr(RPX * (1 + EEX) * CMUX).

e. Substituting (d) into (c) above yields

GAMAX = arccos((sqr(RPX * (1 + EEX) * CMUX) / (RRX * VELX))

f. Substituting from (2) above yields

GAMAX = arccos(sqr((RPX * (1 + EEX)) / (RRX * QQX))).

10. Calculate the perigee velocity

VPX = sqr(CMUX * (1 - EEX) / RPX).

11. Calculate the circular velocity at perigee

VCIRX = sqr(CMUX / RPX).

12. Calculate the true anomaly

THETAX = arccos(((PPX / RRX) - 1) / EEX)

13. Calculate the eccentric anomaly

AEX = arccos((EEX + cos(THETAX)) / (1 + EEX * cos(THETAX))).

14. Compare the orbital radius at LP (RRX) to the apogee radius (AAX * (1 + EEX)). If the orbital radius at LP is greater than the apogee radius, print a message on the screen indicating the difference in nautical miles. Also display the following:

a. LP latitude (ALAT)
b. LP longitude (ALON)
c. vis-viva parameter (QQX)
d. semi-major axis (AAX)
e. eccentricity (EEX)
f. flight path angle (GAMAX)
g. perigee velocity (VPX).

15. Calculate the time of flight.

a. If the orbit is elliptical:

TIMX = sqr(AAX^3/CMUX) * (AEX - EEX * sin(AEX)).

b. If the orbit is hyperbolic:

TIMX = sqr(-AAX^3/CMUX)*(EEX*sinh(EEX) - log(cosh(EEX) + sinh(EEX))).

Appendix H. Subroutine POSVELMO

The subroutine POSVELMO receives the parameter TIM (number of Julian centuries from the year 2000) and returns the moon's position and velocity at that time. Specifically, it returns the moon's distance from the Earth's center, in Earth radii (RRM); velocity in the x-direction (along the Earth-Moon line), in feet per second (XDLO); and velocity in the y-direction (direction of the moon's orbit), in feet per second (YDLO).

1. Calculate a fraction of time that represents half a second.
   DELT = 0.5 seconds / (36525 days/century * 24 hrs/day * 3600 seconds/hr)
   = 1.58440E-10 centuries.

2. Call the subroutine MOON with the parameter (TIM minus DELTA) to determine the moon's distance in Earth radii at half a second before TIM. This distance is RM1.

3. Call the subroutine MOON with the parameter (TIM plus DELTA) to determine the moon's distance in Earth radii at half a second after TIM. This distance is RM2.

4. Calculate the velocity of the moon in the -x (radial) direction.
   XDLO = [(RM2 Earth radii - RM1 Earth radii) / (1 sec)] * 20,925,741 ft/radii.

5. Calculate the average radius of the Lunar orbit during the one second centered on TIM.
   RRM = (RM1 + RM2) / 2.

6. Determine the radius of the Lunar orbit from the Earth-Moon barycenter.
   RRMB = RRM - 0.7412789 Earth radii.

7. Calculate the moon's velocity in the direction of its orbit.

   a. Moon's apogee (Apo) = 62.83308 Earth radii.

   b. Moon's perigee (Per) = 55.68264 Earth radii.

   c. Eccentricity (e) = (Apo - Per) / (Apo + Per) = 0.06033.

   d. Semi-latus rectum (p) = Apo(1 -e^2) = 62.60439 Earth radii
      = 1,310,038,967 feet.

   e. Earth's gravitational parameter (mu) = 1.407646822E+16 ft^3/sec^2.

   f. Angular momentum (h) = sqr(mu * p)
      = 4.29427E+12 ft^2/sec * (1 earth radii / 20,925,672.57 ft)
      = 205,215.4 ft*Earth radii/second.

   g. Y-velocity (YDLO) = h / RRMB = 205,215.4/RRMB.

Appendix I. Subroutine MOON

The subroutine MOON receives the parameter T (number of Julian centuries from the year 2000) and returns the approximate location of the moon in geocentric coordinates at that time. Specifically, it returns the right ascension of the moon (RAM), declination of the moon (DECM), and distance to the moon in Earth radii (RM). The formulae are from The Astronomical Almanac of the Year 1984, page D46. All degrees are converted to radians with the conversion factor $C = \pi/180$.

1. Calculate the ecliptic coordinates of the moon.

a. Moon's ecliptic longitude

$$
\begin{aligned}
LAM = {} & 218^\circ.32 + 481267^\circ.833T \\
& + 6^\circ.29 * \sin(134^\circ.9 + 477198^\circ.85T) \\
& - 1^\circ.27 * \sin(259^\circ.2 - 413335^\circ.38T) \\
& + 0^\circ.66 * \sin(235^\circ.7 + 890534^\circ.23T) \\
& + 0^\circ.21 * \sin(269^\circ.9 + 954397^\circ.70T) \\
& - 0^\circ.19 * \sin(357^\circ.5 + 35999^\circ.05T) \\
& - 0^\circ.11 * \sin(186^\circ.6 + 966404^\circ.05T)
\end{aligned}
$$

b. Moon's ecliptic latitude

$$
\begin{aligned}
BETA = {} & 5^\circ.13 * \sin(93^\circ.3 + 483202^\circ.03T) \\
& + 0^\circ.28 * \sin(228^\circ.2 + 960400^\circ.87T) \\
& - 0^\circ.28 * \sin(318^\circ.3 + 6003^\circ.18T) \\
& - 0^\circ.17 * \sin(217^\circ.6 - 407332^\circ.20T)
\end{aligned}
$$

c. Horizontal parallax

$$
\begin{aligned}
PIE = {} & 0^\circ.9508 \\
& + 0^\circ.0518 * \cos(134^\circ.9 + 477198^\circ.85T) \\
& + 0^\circ.0095 * \cos(259^\circ.2 - 413335^\circ.38T) \\
& + 0^\circ.0078 * \cos(235^\circ.7 + 890534^\circ.23T) \\
& + 0^\circ.0028 * \cos(269^\circ.9 + 954397^\circ.70T)
\end{aligned}
$$

d.Semi-diameter of moon's orbit

$$SD = 0.2725 * PIE$$

e. Distance to the moon in Earth radii

$$RM = 1 / \sin(PIE)$$

2. Form the geocentric direction cosines to rotate into geocentric coordinates.

a. $l = \cos(BETA)\cos(LAM)$
b. $m = 0.9175*\cos(BETA)\sin(LAM) - 0.3978*\sin(BETA)$
c. $n = 0.3978*\cos(BETA)\sin(LAM) + 0.9175*\sin(BETA)$
where $l = \cos(DECM)\cos(RAM)$, $m = \cos(DECM)\sin(RAM)$, $n = SIN(DECM)$.

3. Then:

   a. RAM = arctan(m/l) [right ascension]
   b. DECM = arcsin(n) [declination]

The errors will rarely exceed 0.2 Earth radii in distance (RM), 0.3° in right ascension (RAM), and 0.2° in declination.

Appendix J. Subroutine VELTRANS

The subroutine VELTRANS converts an orbital velocity vector into rectangular coordinates (see figure K-5). Parameters received by the subroutine are velocity (VEL), flight path angle (GAMA), azimuth (AZM), latitude above the Earth-Moon plane (ALAT), and longitude from the Earth-Moon line (ALON). A set of intermediate calculations is performed to express the velocity vector in terms of a radial component, a latitudinal component, and a longitudinal component. Each of these three components is further resolved into x-, y-, and z-components. Finally, all three x-components, all three y-components, and all three z-components are summed to provide the total x-, y-, and z-components of velocity.

1. Conversion of velocity vector into spherical coordinates.

From the geometry, the radial component of velocity, R, is calculated to be
VEL * sin(GAMA). (See figure K-6).
The component along the orbital path, R , is
VEL * cos(GAMA).
This orbital path component of velocity is further resolved into a latitude component, LAT, and a longitude component, LON (see figure K-7). Again, from the geometry,
LON = R * sin(AZM) and
LAT = R * cos(AZM).

2. Conversion of spherical coordinates into rectangular coordinates.

a. Conversion of radial component into rectangular coordinates.

Refer to figure K-8. The projection of R onto the x-y plane is
R * cos(LAT).
The negative x-component of this is
R * cos(LAT) * cos(LON)
so the x-component, X, is
-R * cos(LAT) * cos(LON).
The negative y-component of R is
R * cos(LAT) * sin(LON)
so the y-component, Y, is
-R * cos(LAT) * sin(LON).
The z-component, Z, is
R * sin(LAT).

b. Conversion of latitude component into rectangular coordinates.

Refer to figures K-9 and K-10. LAT is perpendicular to the radial vector, R. A line in the z-direction that meets the tip of LAT and intersects the radius vector produces the angles a and b, where
b = 90 - LAT and
a + b + 90 = 180. Therefore,
a = LAT.
From the geometry, the z-component of LAT, ZLAT, is
LAT * cos(LAT).

The projection of LAT onto the x-y plane is
LAT * sin(LAT). (see figures K-11).
The x-component of this, XLAT, is
LAT * sin(LAT) * cos(LON).
The y-component of this, YLAT, is
LAT * sin(LAT) * sin(LON).

c. Conversion of longitude component into rectangular coordinates.

Refer to figures K-12 and K-13. LON is always parallel to the x-y plane, so the z-component of LON, ZLON, is always zero. Using the same proof as in (b) above, it can be seen that the angle between LON and the y-component of LON is equal to LON. From the geometry, the x-component of LON, XLON, is
LON * sin(LON).
The negative y-component of LON is
LON * cos(LON),
so the y-component, YLON, is
-LON * cos(LON).

3. Sum of the rectangular coordinates.

All of the x-, y-, and z-components are summed to provide the complete rectangular coordinates of the velocity vector.
VXX = X + XLAT + XLON
VYX = Y + YLAT + YLON
VZX = Z + ZLAT + ZLON.

The projection of LAT onto the x-y plane is
LAT * sin(LAT). (see figures K-11).
The x-component of this, XLAT, is
LAT * sin(LAT) * cos(LON).
The y-component of this, YLAT, is
LAT * sin(LAT) * sin(LON).

c. Conversion of longitude component into rectangular coordinates.

Refer to figures K-12 and K-13. LON is always parallel to the x-y plane, so the z-component of LON, ZLON, is always zero. Using the same proof as in (b) above, it can be seen that the angle between LON and the y-component of LON is equal to LON. From the geometry, the x-component of LON, XLON, is
LON * sin(LON).
The negative y-component of LON is
LON * cos(LON),
so the y-component, YLON, is
-LON * cos(LON).

3. Sum of the rectangular coordinates.

All of the x-, y-, and z-components are summed to provide the complete rectangular coordinates of the velocity vector.
VXX = X + XLAT + XLON
VYX = Y + YLAT + YLON
VZX = Z + ZLAT + ZLON.

Appendix K. Figures

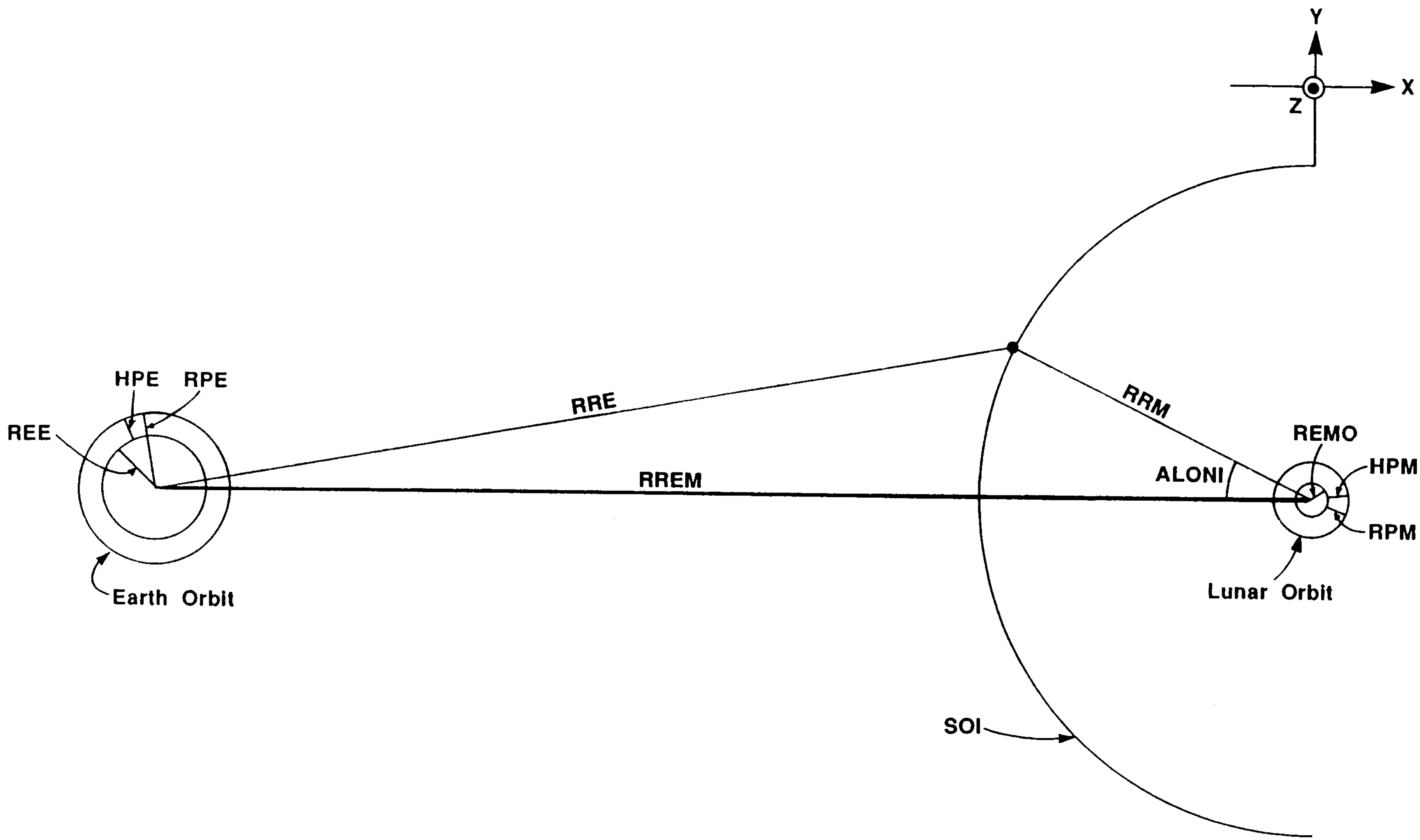

Y
X
Z
HPE
RPE
REE
RRE
RRM
REMO
HPM
ALONI
RREM
RPM
Earth Orbit
Lunar Orbit
SOI

Figure K-1

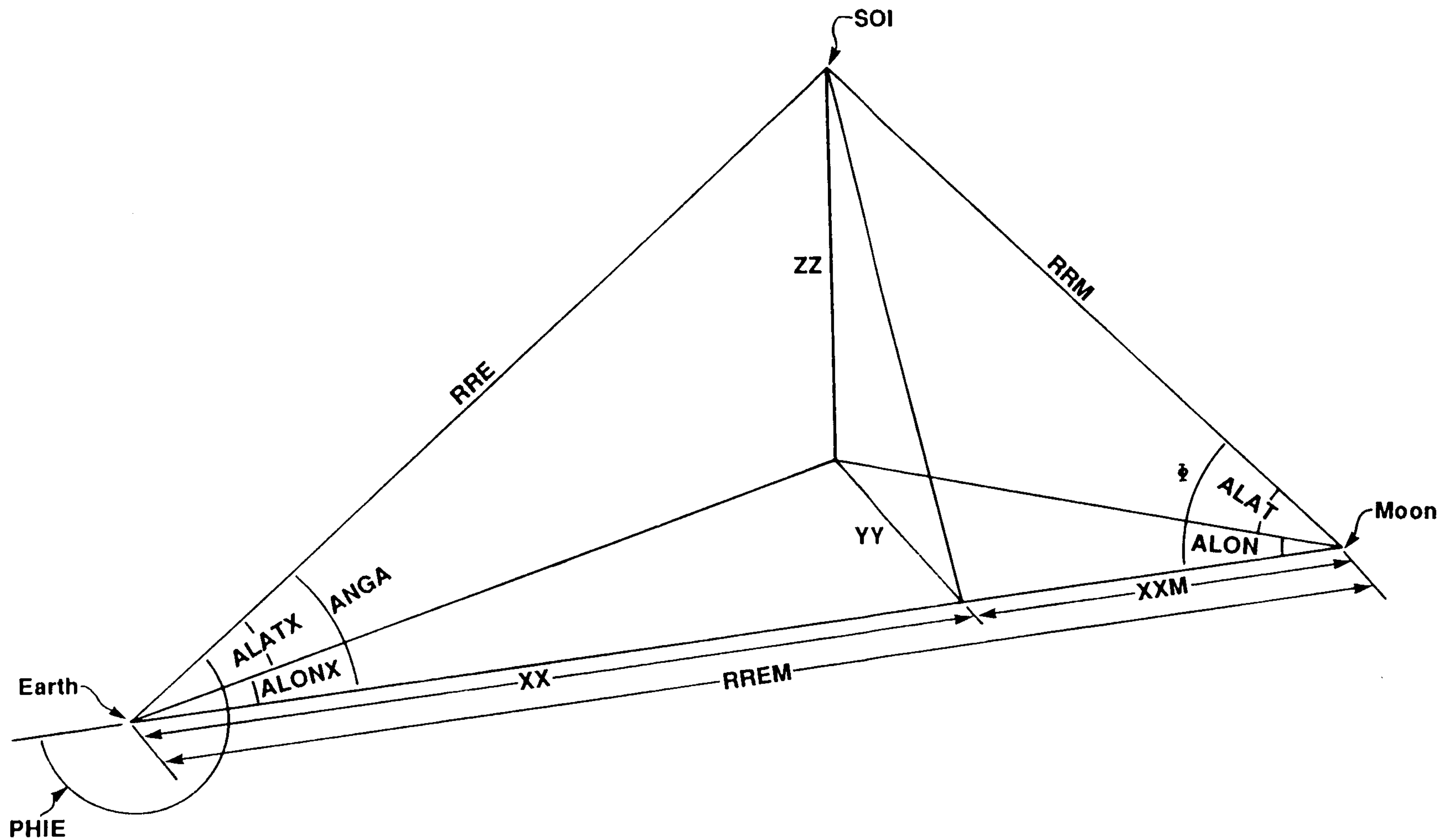

SOI
ZZ
RRM
RRE
Φ
ALAT
Moon
ALON
YY
ANGA
XXM
ALATX
ALONX
Earth
XX
RREM
PHIE

Figure K-2

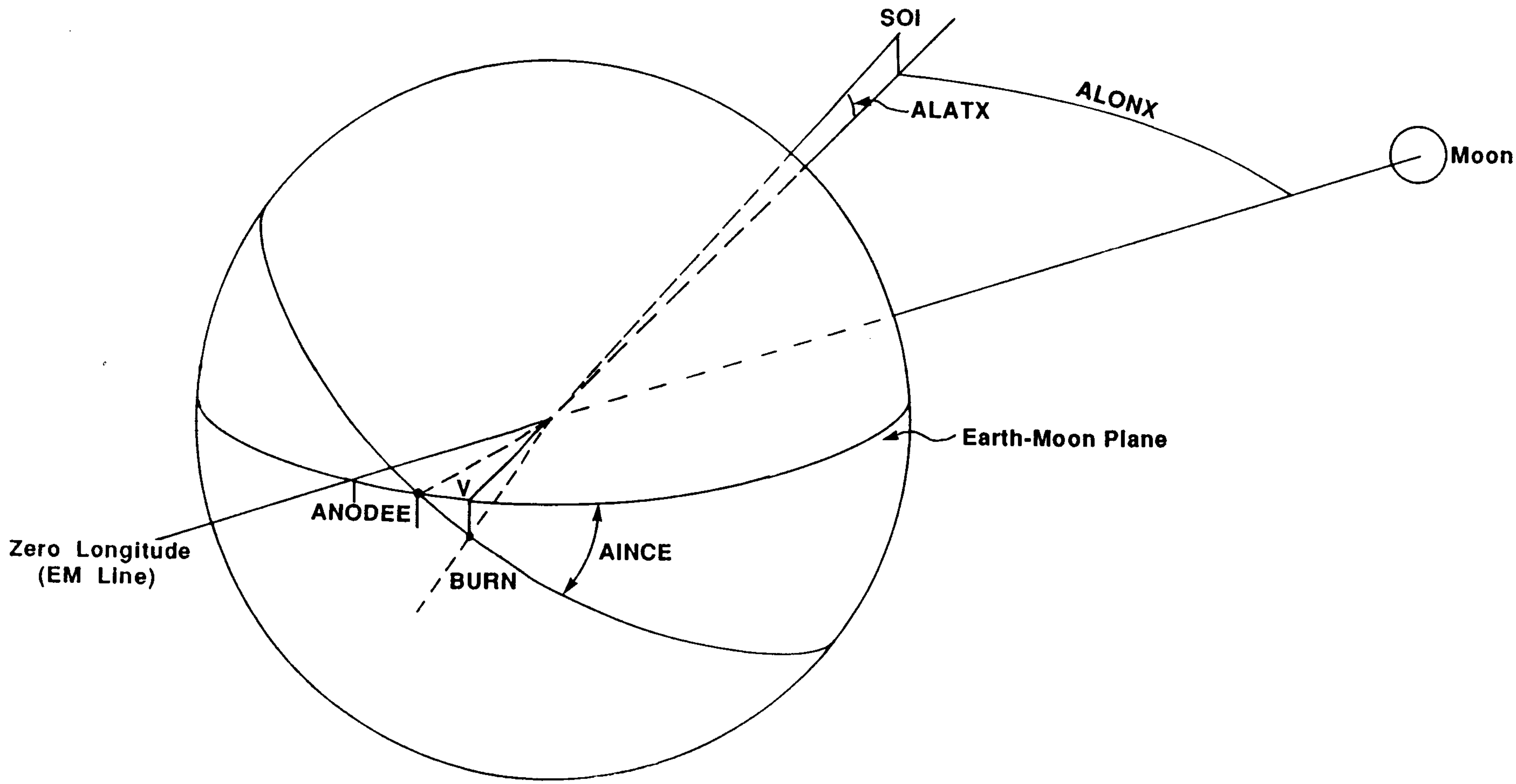

SOI
ALATX
ALONX
Moon
Earth-Moon Plane
V
ANODEE
Zero Longitude
(EM Line)
AINCE
BURN

Figure K-3

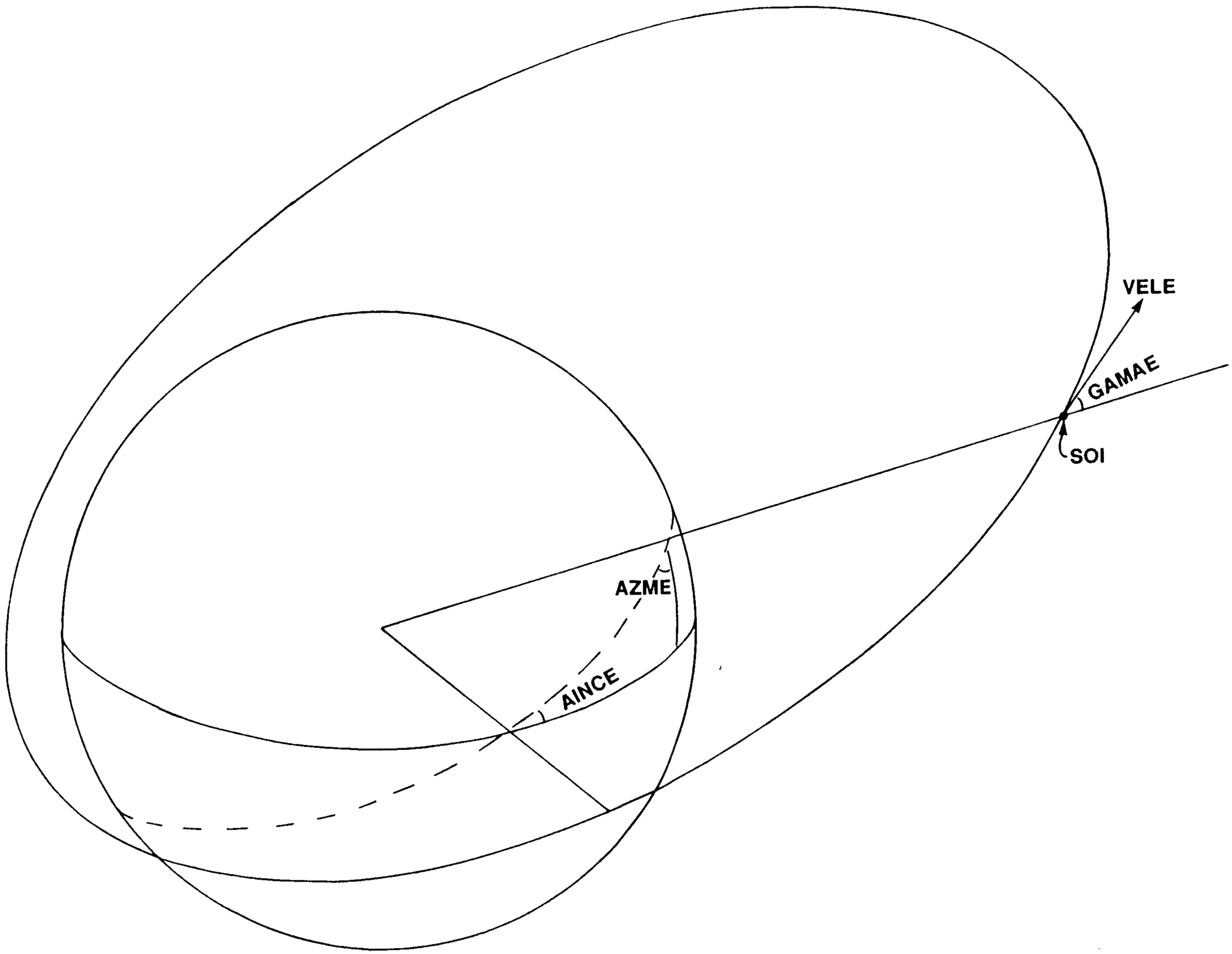
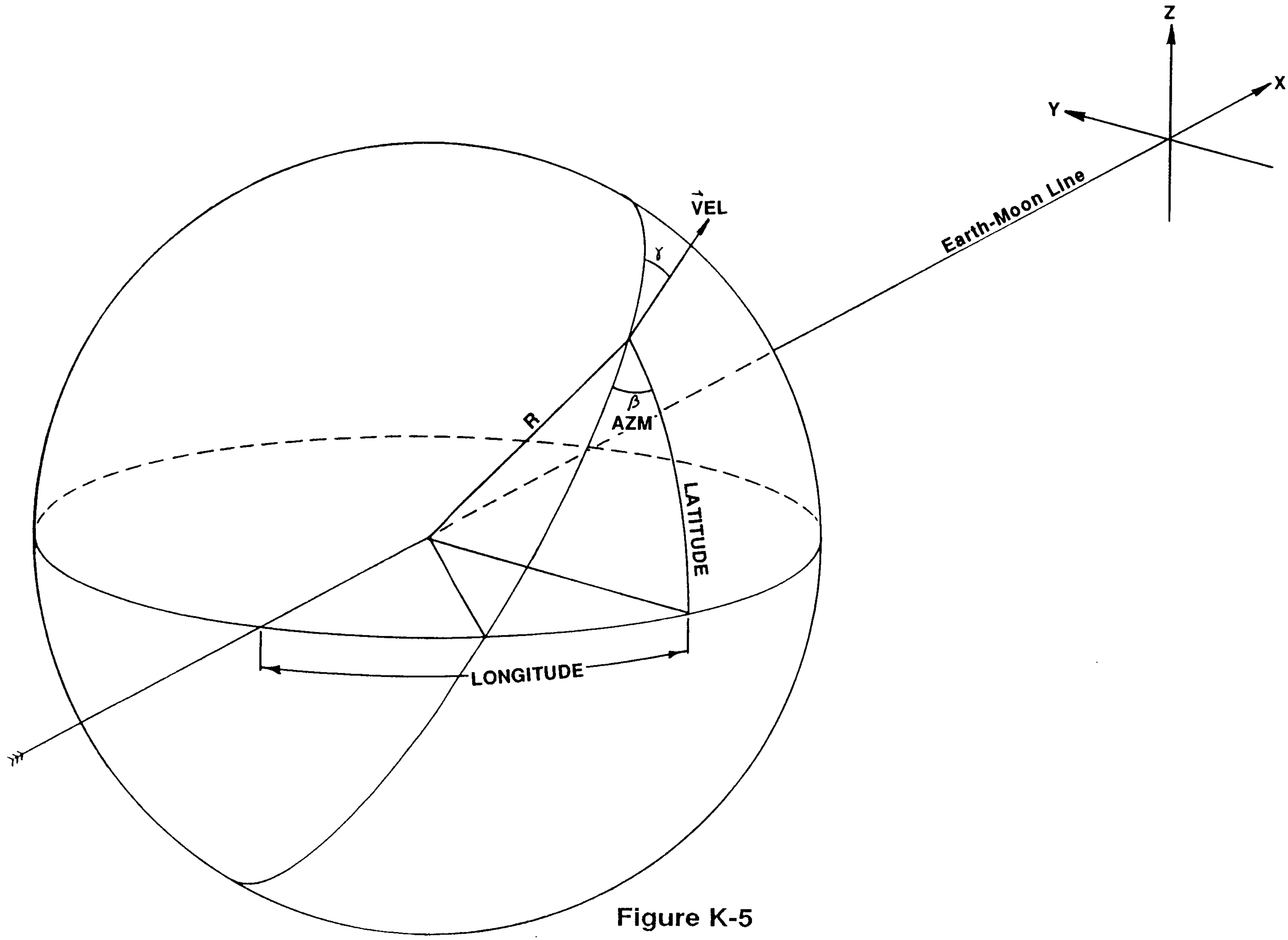


Figure K-4



Z
X
Y
Earth-Moon Line
$\vec{VEL}$
γ
β
AZM
R
LATITUDE
LONGITUDE

Figure K-5

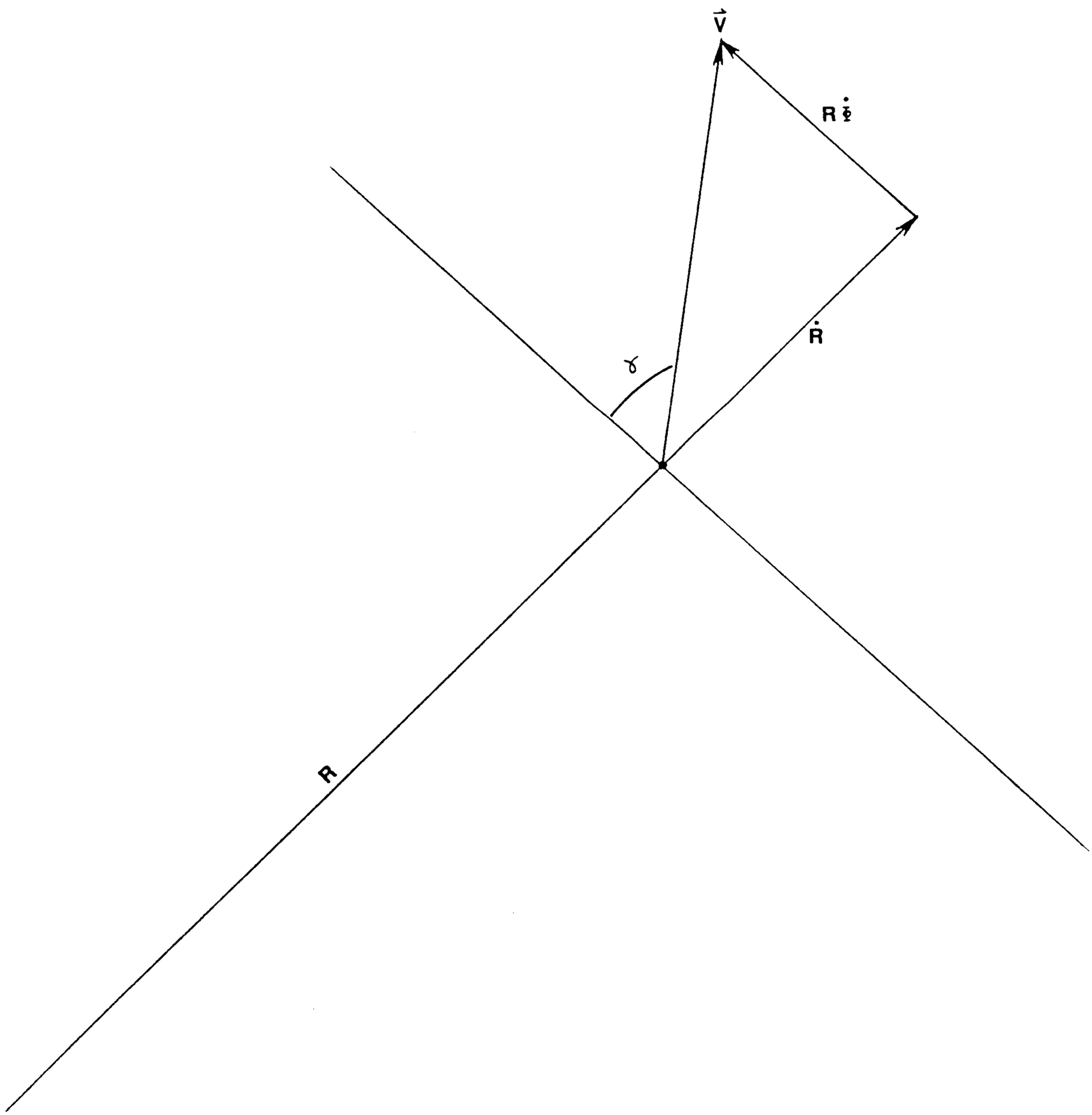


Figure K-6

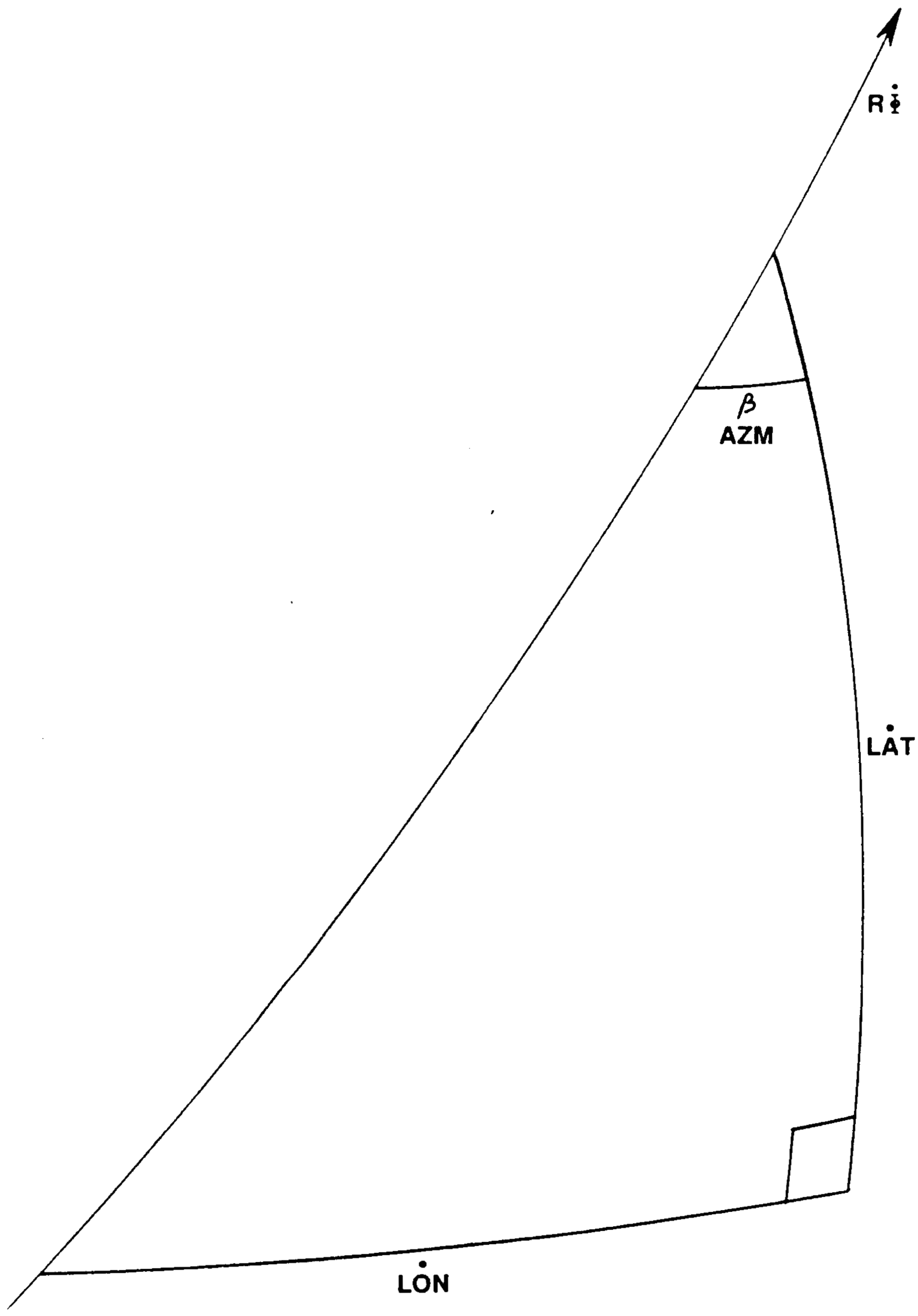


Figure K-7

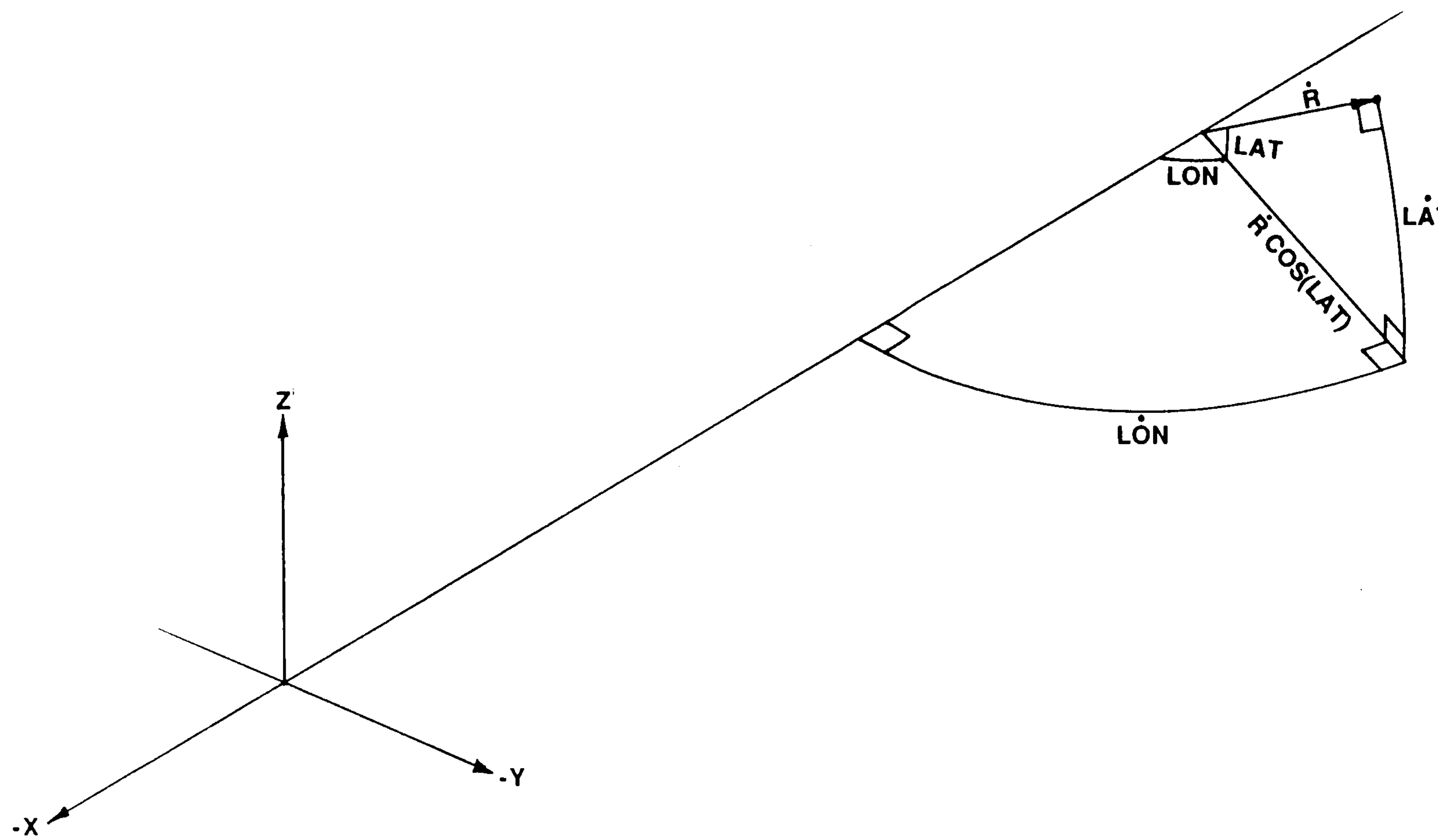


Figure K-8

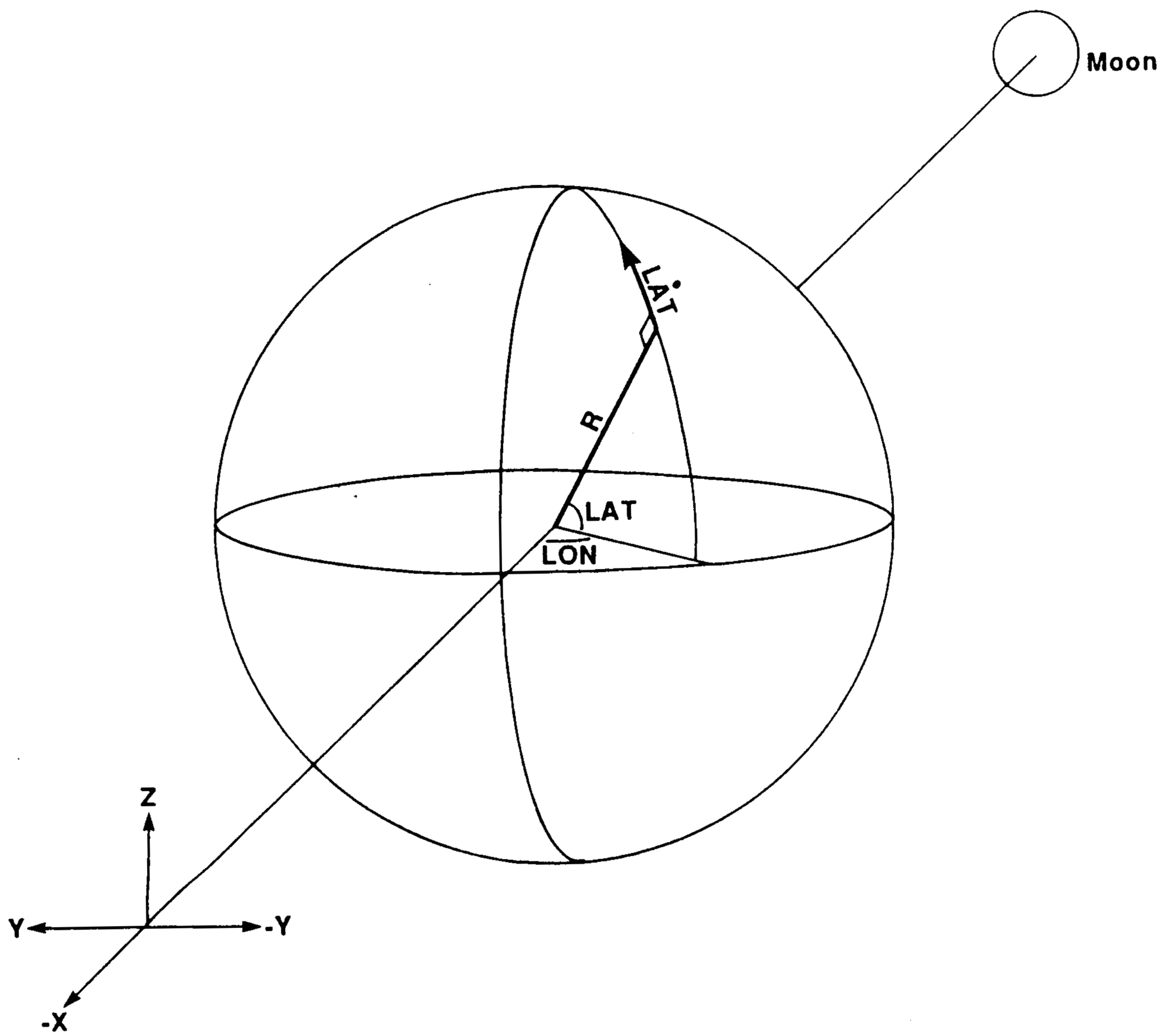


Figure K-9

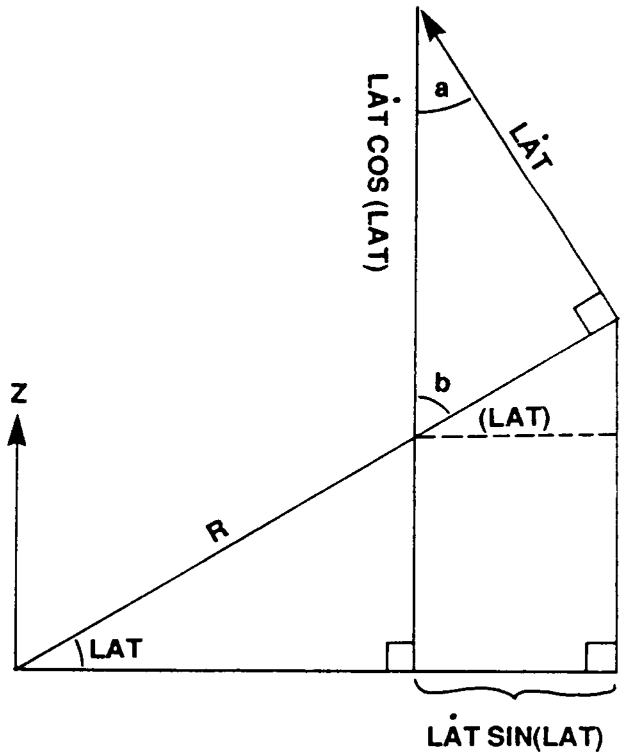


Figure K-10

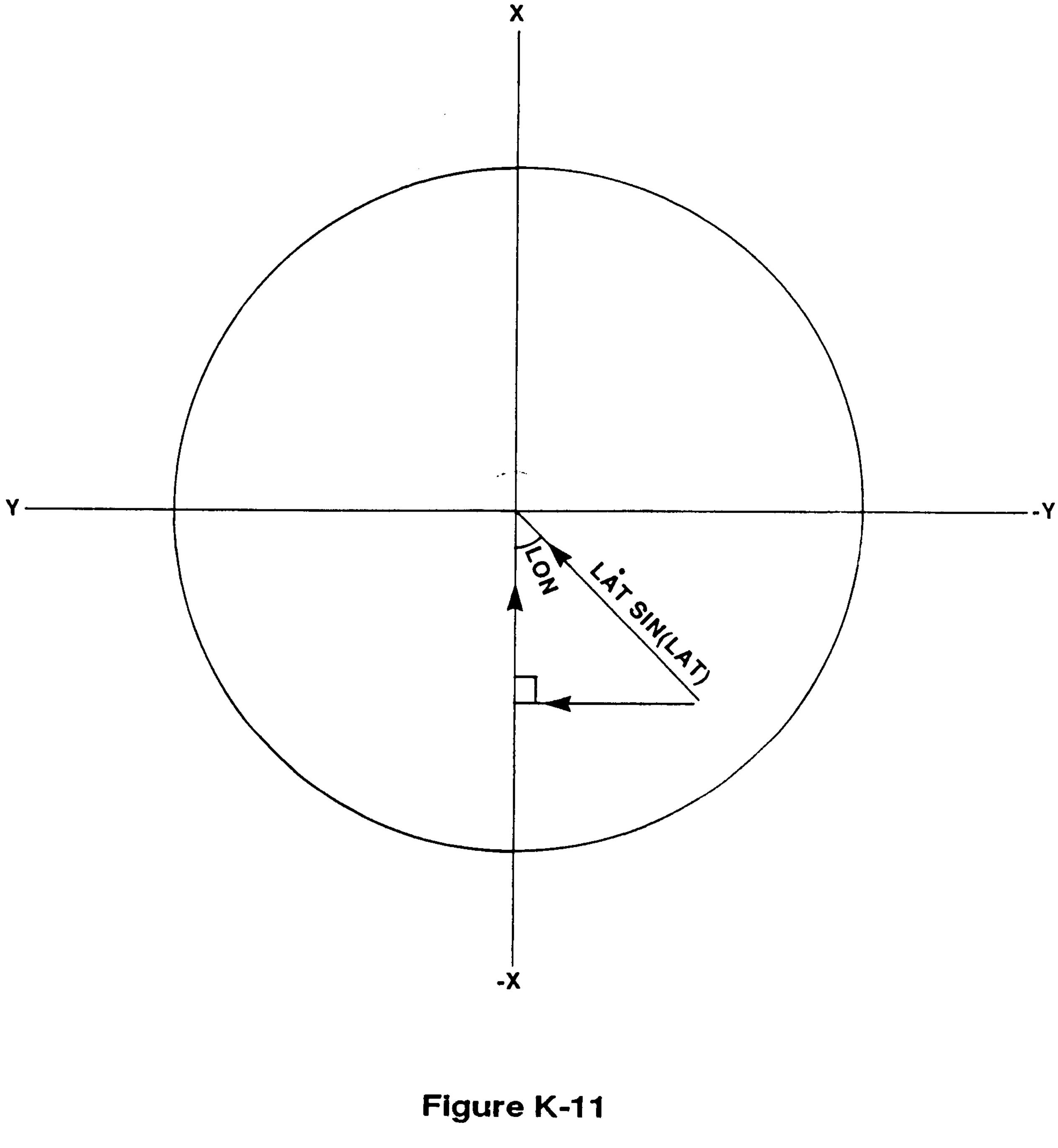


Figure K-11

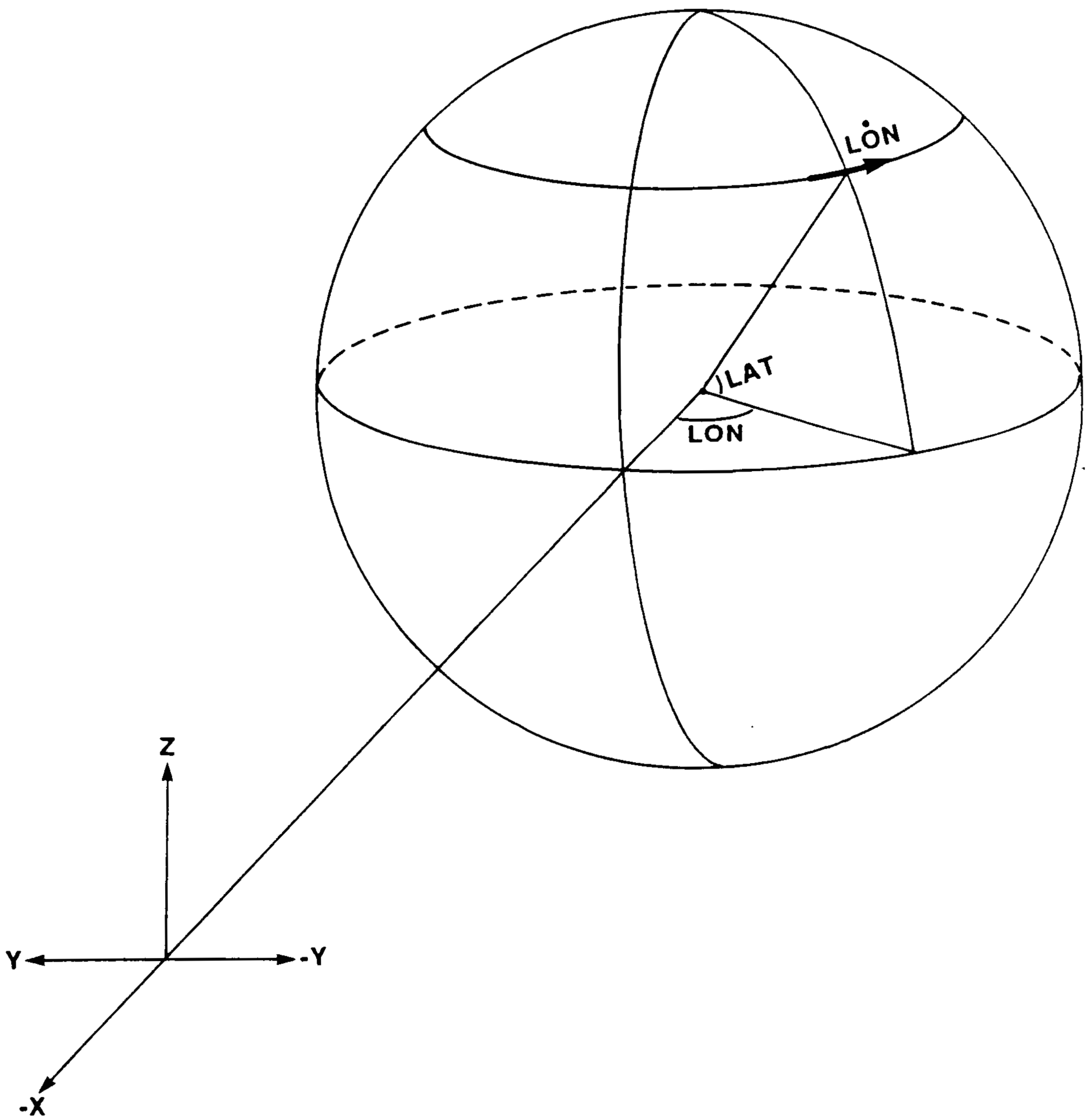


Figure K-12

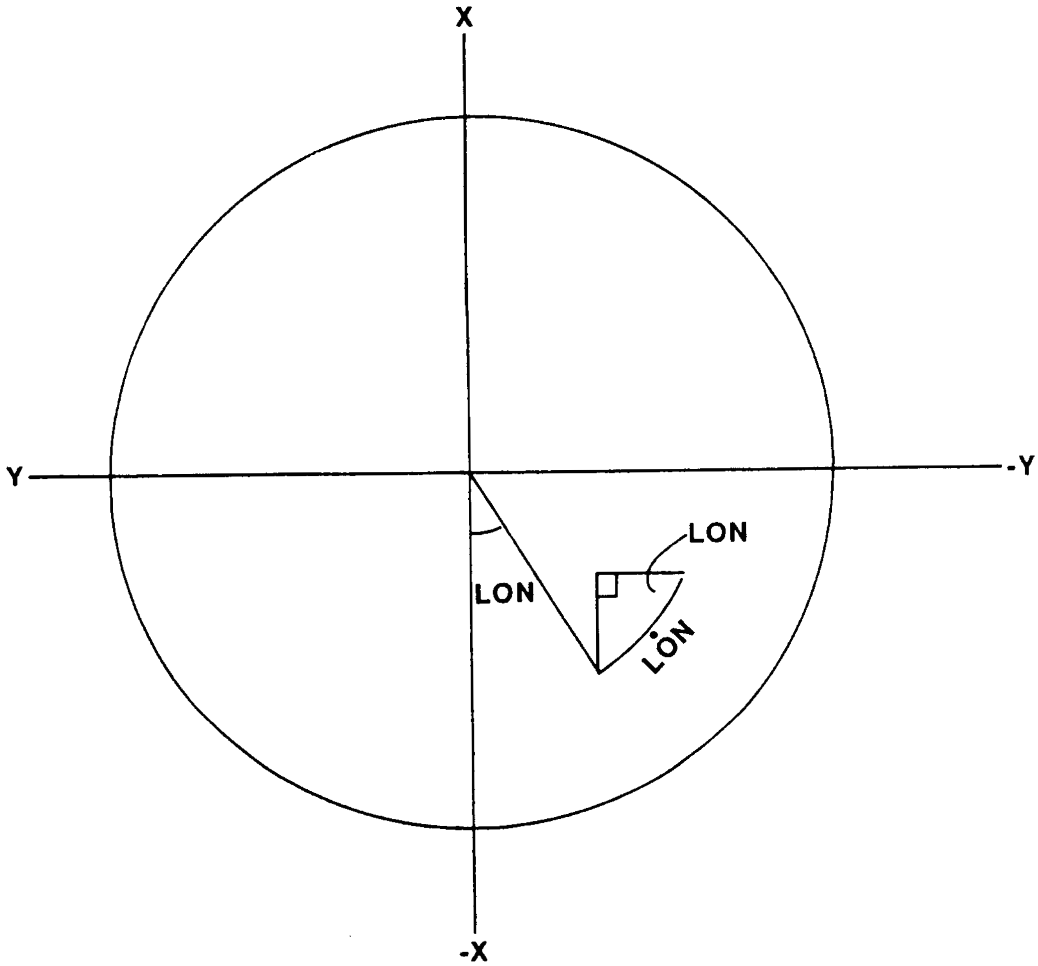


Figure K-13